Midterm Examination

Intermediate Microeconomics (Honors)

Fall 1996

Name:

Carefully read each question before answering. Show all intermediate steps used in arriving at a conclusion. Clearly indicate your final response to each answer.

Part I. True, False, or Uncertain. Write T, F, or U for the response you believe most accurately characterizes the validity of the statement. Give a short but detailed justification of your response in the space provided. Use mathematical, graphical, or verbal arguments as appropriate. Each question is worth 6 points.

1. An individual has an utility function defined over income which is given by 
\[ U(I) = 15I - 2I^2 + I^3/3. \] The individual will refuse all fair bets.

2. The utility function \( U(X, Y) \) has an associated marginal rate of substitution given by
\[ (1 + Y)/(2 + X). \] \( U \) is homothetic.

3. An individual with an utility function \( U(X, Y) \) spends half of her income on \( X \) and half on \( Y \). If the income elasticity of \( X \) is greater than 2 then \( Y \) is an inferior good.
4. An individual has utility defined over consumption of a market good \( X \) with price 1 and leisure. Her utility function is given by \( .5 \ln(X) + .5 \ln(L) \), where \( L \) denotes leisure. Her time endowment (\( T \)) is equal to 1 and her nonlabor income (\( I \)) is 4. If she is offered a wage of 5 she will supply time to the labor market.

5. An individual has an utility function defined over two goods \( X \) and \( Y \) which is given by
   \[ U(X, Y) = \min(X, Y). \]
   Denote the individual’s income level by \( I \) and the prices of the two goods by \( p_X \) and \( p_Y \). The individual’s indirect utility function is given by \( I/(p_X + p_Y) \).

6. In a population of \( N \) individuals population member \( i \) has an utility function given by
   \[ U_i(X, Y) = a_iX + Y \]
   where \( 0 < a_i < 1 \), \( i = 1, ..., N \). The income of population member \( i \) is equal to \( I_i \). If the price of good \( X \) is equal to the price of good \( Y \), total market demand only depends on total income in the population \( \text{[i.e., } \sum_{i=1}^{N} I_i] \).

7. Let demand for a good \( X \) by an individual be given by \( X^*(p_X, p_Y, I) \) where \( p_X \) and \( p_Y \) are the prices of the two goods consumed by the individual and \( I \) is the individual’s income.
   \[ X^*(p_X, p_Y, I) = X^*(a p_X, a p_Y, a I) \]
   for all \( a > 0 \).

Part II. Problems.
Answer each part of each of the following problems. Remember to show all of your work.

8. (18 points) Mitzi has the following utility function defined over consumption (\( c \)) and leisure (\( l \)):
   \[ U(c, l) = 10c + 50l - l^2. \]
The price of consumption is 1 and Mitzi’s nonlabor income is $I = 20$. Mitzi’s time endowment ($T$) is equal to 10.

a. Find Mitzi’s reservation wage.

b. If Mitzi is offered a wage of 4, determine how much time she will spend in the market.

c. What is the effect of a tax on nonlabor income on labor supply in this model?
9. (20 points) An individual has $I$ dollars and can invest some portion of it in a risky investment. With probability .5 the investment turns out well (the good state) and with probability .5 it turns out badly (the bad state). The individual’s utility function is $U(W) = \ln(W)$, where $W$ is his final wealth level. Let $x$ denote the amount he invests in the risky asset.

a. If the rate of return on the investment in the good state ($r_g = 1$ and the rate of return in the bad state ($r_b = -2/3$), find his optimal investment ($x$).

b. The investor happens upon another investment which has $r_g = .5$ and $r_b = -1/3$. If he invests his money here what is his optimal investment level?

c. Which of the two investment opportunities has the highest expected utility associated with it? You can answer this by comparing the average return and the variance in returns associated with the two investments taking into account the individual’s attitude toward risk.
10. (20 points) An individual has an utility function given by $U(X, Y) = \ln(X) + Y$.
   a. If $p_X = 1$, $p_Y = 4$, and $I = 3$, solve for the individual’s utility-maximizing demand for $X$.

   b. Find the demand for $X$ when the price of $X$ increases to 2 [the price of $Y$ and income remain 4 and 3, respectively].

   c. How much of the change in demand when the price of $X$ changes from 1 to 4 is due to the substitution effect and how much is due to the income effect?