Carefully read each question before answering. If a question seems ambiguous, clearly state your interpretation of it before answering. Show all intermediate steps used in arriving at a conclusion. Clearly indicate your final response to each answer.

Part I. True, False, or Uncertain. Write T, F, or U for the response you believe most accurately characterizes the validity of the statement. Give a short but detailed justification of your response in the space provided. Use mathematical, graphical, or verbal arguments as appropriate. Each question is worth 6 points.

1. An individual has a utility function given by \( U(x_1, x_2) \), with \( \frac{\partial U(x_1, x_2)}{\partial x_1} > 0 \) and \( \frac{\partial U(x_1, x_2)}{\partial x_2} < 0 \) for all values of \( x_1 \) and \( x_2 \). The individual’s demand for \( x_1 \) will be independent of the price of \( x_2 \).

2. If an individual has a utility function given by \( U(x_1, x_2) = \alpha \ln(x_1) + \beta \ln(x_2) \), with \( \alpha > 0 \) and \( \beta > 0 \), then neither \( x_1 \) nor \( x_2 \) can be a luxury good.

3. If a good has a compensated substitution elasticity of \(-.5\) and an income elasticity of \(-1\), it is a Giffen good.
4. An individual with a utility function defined over consumption given by $U(C) = -1/C$ will never accept a fair bet.

5. An individual with a utility function given by $U(x_1, x_2) = x_1 + \ln(x_2)$ will always consume a positive amount of $x_2$.

6. An individual has a utility function defined over the consumption of a market good ($c$) and leisure ($l$), where $U(c, l) = .5 \ln(c) + .5 \ln(l)$. If the individual earns no more than 100 dollars, the tax rate on her earnings is .2, while every dollar of earnings beyond the 100th is taxed at the rate .5. If the individual is paid a (gross) wage of 5 per hour, and if she works, she will never choose to work 20 hours.
Part II. Problems.
Answer each part of each of the following problems. Remember to show all of your work.

1. (18 points) A population of $N$ individuals is evenly divided between type $A$ and type $B$ individuals. These people are all otherwise identical except for the fact that type $A$ individuals have no nonlabor income ($I_A = 0$) while type $B$ individuals have nonlabor income $I_B = 5$. All individuals in the population have a utility function defined over a consumption good $X$ and leisure $l$ which is given by $U(X, l) = .5 \ln(X) + .5 \ln(l)$. Each person’s time endowment ($T$) is equal to 1 and the price of $X$ is equal 1.

a. Find the reservation wage of a type $A$ individual, and then find the reservation wage of a type $B$ individual.

b. If the market wage rate is 5 [that is, the wage rate offered to all individuals] determine the total amount of labor supplied to the market.

c. If the government imposes a tax rate of .5 on nonlabor income only, determine the total amount of time supplied to the market.
2. (16 points) An individual with a wealth endowment of 10 ($W$) has a utility function defined with respect to income which is $U(I) = \ln(I)$. He has the opportunity to purchase a risky asset. Given that $Y$ units of the risky asset are purchased, in the good state of the world his total income will be $W + r_g Y$, and in the bad state of the world his total income will be $W + r_b Y$. Let the probability of the good state of the world be .5 and the probability of the bad state be .5.

a. If $r_g = 1$ and $r_b = -.5$, how much of the risky asset will the individual buy so as to maximize expected utility?

b. If the net rate of return in the bad state changes from $-.5$ to $-1$, how much will the individual invest?
3. (30 points) An individual lives for two periods and seeks to maximize her expected lifetime income. Her income is 0 in each period she doesn’t participate. If she searches for a job, she pays a cost of 1 in each period in which she searches. Every period in which she searches she is offered a job. With probability .25 the job offer pays 1 in each period of employment and with probability .75 it pays 5 in every period of employment. If the individual accepts a job in the first period she keeps the same job in period 2. Jobs begin in the period in which they are accepted. Offers rejected in period 1 (that is, an offer from a particular firm) cannot be accepted in later periods.

a. If the individual enters her second period of life without a job, will she search for one? If so, which wage offers will she accept?

b. Will the individual search for a job in her first period of life? If so, which wage offers will she accept?

c. What is the probability that the individual will be without a job during period 1? What is the probability she will be without a job during period 2?
d. If the cost of search increases to 3 for each period in which search takes place, what proportion of individuals will not be employed during period 1?

e. If the cost of search increases to 9 for each period in which search takes place, what proportion of individuals will not be employed during period 1?