Carefully read each question before answering. If a question seems ambiguous, clearly state your interpretation of it before answering. Show all intermediate steps used in arriving at a conclusion. Clearly indicate your final response to each answer.

Part I. True, False, or Uncertain. Write T, F, or U for the response you believe most accurately characterizes the validity of the statement. Give a short but detailed justification of your response in the space provided. Use mathematical, graphical, or verbal arguments as appropriate. Each question is worth 6 points.

1. An individual has a utility function given by $U(m) = \ln(m) + m - \exp(-m)$, where $m$ denotes money. The individual will refuse all fair bets.

2. An individual has a utility function defined over two goods given by $U(x_1, x_2)$. The demand function for good 1 is given by $x_1^*(p_1, p_2, I) = p_2p_1^{-1}I^\alpha$, where $p_j$ is the price of good $j$ and $I$ denotes income. If $\alpha > 1$, $x_1$ is a luxury good.

3. If a good has a compensated substitution elasticity of $-1$ and an income elasticity of $-1$, it cannot be a Giffen good.
4. If the utility function $U(x_1, x_2)$ is homothetic, then the income elasticity of each good must be equal to 1.

5. An individual with a utility function given by $U(x_1, x_2) = x_1 + \ln(x_2 + 1)$ will always consume a positive amount of $x_2$ for any $p_1 > 0$, $p_2 > 0$, and $I > 0$.

6. An individual has a utility function defined over income, $U(I)$, with $dU(I)/dI > 0$ for all $I > 0$. In the good state of the world, the individual receives an income of 5 and in the bad state of the world the individual receives an income of 10. She is offered actuarially-fairly priced insurance and purchases 3 units of it. Then $d^2U(I)/dI^2 < 0$ for all $I > 0$. 


7. A store runs a promotional sale in which the first 3 units of a good \( (x_1) \) are priced at 3 per unit and all additional units have a price of 5 (assume that the good is perfectly divisible - that is, you could buy 2.913 units if you so desired). If a customer’s utility function is given by \( U(x_1, x_2) = x_1^\alpha x_2^{\frac{1}{2} - \alpha} \), with \( 1 > \alpha > 0 \), and the price of \( x_2 \) is positive, then he will never purchase exactly 3 units of \( x_1 \).

8. An individual has a utility function defined over three goods, \( U(x_1, x_2, x_3) \). The expenditure shares of goods 1 and 2 \( (s_1 \text{ and } s_2) \) are each equal to .25. Both the income elasticity of good 1 \( (\eta_1) \) and good 3 \( (\eta_3) \) are equal to 1.5. The income elasticity of good 2 is -3.
Part II. Problems.
Answer each part of each of the following problems. Remember to show all of your work.

9. (10 points) A population of individuals is evenly divided between type A and type B individuals. Each individual receives an income of $I_g = 5$ in the good state of the world and $I_b = 1$ in the bad state. Both types of individuals have a probability of experiencing the good state of .5. Type A individuals have utility functions given by $U_A(I) = -\exp(-I)$, while type B individuals have utility functions given by $U_B(I) = \exp(I)$. Insurance is available that is priced in an actuarially-fair manner (that is, insurance companies make 0 profits). How much insurance is bought by type A individuals? How much is bought by type B individuals?
10. (21 points) Half of a population of 100 individuals have utility functions given by $U_A(x_1, x_2) = x_1 + x_2$, while the other half have utility functions given by $U_B(x_1, x_2) = \ln(x_1) + \ln(x_2)$. The income of each individual in the population is $I = 1$.

(a) Determine the market demand for $x_1$ if the price of good 1 ($p_1$) is 1 and the price of good 2 ($p_2$) is 2.

(b) The price of the second good decreases to .5 (call it $p'_2$). Determine the market demand for $x_1$ at this new price for $x_2$.

(c) Within the subpopulation of type $A$ individuals, determine the substitution and income effect in the demand for $x_1$ associated with this change in the price of good 2 (use the Marshallian-compensated income measure).
11. (21 points) An individual has a utility function defined over a good purchased in the market, \( x \), and leisure, \( l \). The utility function is given by \( U(x, l) = \ln(l) + x \). Let the price of \( x \), \( p_x \), be equal to 1, the individual’s nonlabor income, \( I \), be equal to 1, and the individual’s total time endowment, \( T \), be equal to 1.

(a) Determine the individual’s reservation wage rate, \( w^* \).

(b) Say that the individual is offered a wage of 2. How much time will she spend in the labor market? How much of the market good will she consume?

(c) Given that the wage rate is equal to 2, and using all of the other values describing the choice set which were given to you above, compute the elasticity of leisure demand with respect to infinitesimal changes in the level of nonlabor income \( (I) \), denoted \( \eta_l \). Also compute the income elasticity of the demand for \( x \) for this case, \( \eta_x \). Does the share-weighted sum of income elasticities equal 1 in this case? If not, why?