Topic 2

Incorporating Financial Frictions in DSGE Models

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Overview

Conventional Model with Perfect Capital Markets:

1. Arbitrage between return to capital and riskless rate

\[ E_t \beta \Lambda_{t,t+1} R_{k,t+1} = E_t \beta \Lambda_{t,t+1} R_{t+1} \]

where \( \beta \Lambda_{t,t+1} \) is the household’s stochastic discount factor

2. Financial structure irrelevant.
Overview (con’t)

With capital market frictions:

1. External finance premium ⇒

\[ E_t \beta \Lambda_{t,t+1} R_{kt+1} > E_t \beta \Lambda_{t,t+1} R_{t+1} \]

2. Premium depends inversely on borrower balance sheets ⇒

3. If borrower balance sheets move procyclically, external finance premium move countercyclically:

⇒ feedback between financial and real sectors ("financial accelerator,"")
⇒ disturbances originating in the financial sector can have real effects.
Bernanke/Gertler/Gilchrist Financial Accelerator Model

Dynamic General Equilibrium Framework with

1. Money
2. Imperfect Competition
3. Nominal Price Rigidities (Calvo staggered price setting.)
Sectors

1. Households

2. Business Sector
   (a) entrepreneur/firms
   (b) capital producers
   (c) retailers

3. Central Bank
Households

- Objective

\[
\max E_T \sum_{i=0}^{\infty} \beta^i \log (C_{t+i}) + a_m \log \left( \frac{M_{t+i}}{P_{t+i}} \right) - a_n \frac{1}{1 + \gamma_{n}} L_{t+i}^{1+\gamma_{n}}
\]

subject to

\[
C_t = \frac{W_t}{P_t} L_t + \Pi_t - T_t - \frac{M_t - M_{t-1}}{P_t} - \frac{1}{1 + i_t} B_t - B_{t-1}
\]

- As in Woodford (2003), we restrict attention to the cashless limit of the economy (the limit as \( a_m \to 0 \)).
Decision Rules

- labor supply

\[
\frac{W_t}{P_t} = a_n L_{t+i}^{\gamma_n} / \left( \frac{1}{C_t} \right)
\] (3)

- consumption/saving;

\[
\frac{1}{C_t} = E_t \left\{ (1 + i_t) \frac{P_t}{P_{t+1}} \beta \frac{1}{C_{t+1}} \right\}
\] (4)
Entrepreneurs/Firms

- Produce wholesale output

- Competitive, risk neutral, face capital market frictions.

- A measure unity in the market at any time.

- i.i.d survival probability $\theta$: The expected horizon is accordingly $\frac{1}{1-\theta}$. $1 - \theta$ enter to replace exiting entrepreneurs.

- Exiting entrepreneurs make a small transfer to new entrepreneurs and then consume the rest.
Production Technology

The production technology is given by

\[ Y_t = \omega_t A_t (K_t)^\alpha (L_t)^{(1-\alpha)}. \]  \hspace{1cm} (5)

where \( \omega_t \) is i.i.d with

\[ E\{\omega_t\} = 1 \]
Labor Demand

F.O.N.C.

\[ \frac{W_t}{P_{wt}} = (1 - \alpha) \frac{Y_t}{L_t} \]
Capital Demand

- Gross Return to Capital

\[
E_t \{ R_{k_{t+1}} \} = E_t \left\{ \frac{P_{w+1} \alpha Y_{t+1}}{P_{t+1} K_{t+1}} + (1 - \delta) Q_{t+1} \right\} \frac{Q_t}{Q_t}
\]

- Opportunity Cost

\[
E_t \left\{ (1 + i_t) \frac{P_t}{P_{t+1}} \right\}
\]
Capital Demand (con’t)

Under perfect markets, capital demand given by

\[
E_t \{ R_{kt+1} \} = E_t \left\{ (1 + \dot{\epsilon}_t) \frac{P_t}{P_{t+1}} \right\}
\]

With imperfect markets:

\[
E_t \{ R_{kt+1} \} > E_t \left\{ (1 + \dot{\epsilon}_t) \frac{P_t}{P_{t+1}} \right\}
\]
Capital Demand (con’t)

The finance of capital is divided between net worth and debt:

\[ Q_t K_{t+1} = N_t + \frac{B_t}{P_t}. \]
Costly State Verification

Assume:

(i) costly state verification and limited liability

(ii) one period contracts

(iii) payouts based only on firm-specific contingencies

→:

1. Debt with costly default is optimal

2. Agency costs of external finance (expected default costs)

3. Collateral reduces expected default costs
Optimal Choice of Capital

\[ Q_t K_{t+1} = v\left(\frac{E_t \{ R_{kt+1} \}}{E_t \left\{ (1 + i_t) \frac{P_t}{P_{t+1}} \right\}}\right) N_t \]
Optimal Choice of Capital (con’t)

Aggregate Demand for Capital (Inverting the previous equation)

\[ E_t \{ R_{kt+1} \} = (1 + \chi_t) E_t \left\{ (1 + i_t) \frac{P_t}{P_{t+1}} \right\} \]

with

\[ \chi_t = \chi \left( \frac{Q_t K_{t+1}}{N_t} \right) \]

and

\[ \chi'(\cdot) > 0, \; \chi(0) = 0, \; \chi(\infty) = \infty \]
Evolution of Net Worth

\[ N_t = \theta V_t + (1 - \theta)D \]

where

\[ V_t = (1 - m_t)R_{kt}Q_{t-1}K_t - \left[ (1 + i_{t-1}) \frac{P_{t-1}}{P_t} \right] \frac{B_t}{P_{t-1}} \]

with

\[ R_{kt} = \frac{P_w}{P_t} \alpha \frac{Y_{tt}}{K_{tt}} + (1 - \delta)Q_t \]

\[ m_t = \mu G(\omega^*_{t-1}) \]
Evolution of Net Worth (con’t)

• Main Sources of Net Worth Fluctuations
  
  Unexpected movements in $Q_t$ and $P_t$

• Irving Fisher’s debt-deflation hypothesis: unanticipated declines in price level raise real debt burdens.
The Role of Leverage

Given \( Q_{t-1}K_t = N_{t-1} + \frac{B_{t-1}}{P_{t-1}} \)

\[
V_t = \{[(1 - m_t)R_{kt} - R_t]\phi_{t-1} + R_t\}N_{t-1}
\]

with

\[
\phi_{t-1} = \frac{Q_{t-1}K_t}{N_t}
\]

\[
R_t = (1 + i_{t-1}) \frac{P_{t-1}}{P_t}
\]

- The sensitivity of net worth to unanticipated returns is increasing in the leverage ratio \( \phi_{t-1} \).
Capital Producers

- Capital Producers are competitive. They produce new capital and sell at the price $Q_t$.

- Evolution of capital

\[ K_{t+1} = \Phi \left( \frac{I_t}{K_t} \right) K_t + (1 - \delta) K_t \]

\[ \Phi' > 0, \Phi'' < 0, \Phi \left( \frac{I}{K} \right) = \frac{I}{K} \]
Optimal Choice of Investment

\[ E_{t-1}\{Q_t - [\Phi'(\frac{I_t}{K_t})]^{-1}\} = 0 \]

i.e., \( Q \) is increasing \( \frac{I_t}{K_t} \) as in Tobin’s Q theory

Note: Marginal product of capital used in producing new capital goods is zero within a local region of the steady state. See BGG.
Retailers

- Buy wholesale output and sell as differentiated product
- Set prices on a staggered basis as in Calvo (1983)

\[
\frac{P_t}{P_{t-1}} \approx (\mu \frac{P^w_t}{P_t})^\lambda E_t \left( \frac{P_{t+1}}{P_t} \right)^\beta
\]

in loglinear form

\[
\pi_t = \lambda (p_{wt} - p_t) + \beta E_t \pi_{t+1}
\]

- Note: \( p_t - p_{wt} \) is the log price markup.
Resource Constraint

Let $C^e_t \equiv$ entrepreneurial consumption and $M_t \equiv$ total monitoring costs:

$$Y_t = C_t + C^e_t + I_t + G_t + M_t$$

with

$$C^e_t = (1 - \phi)(V_t - D)$$

$$M_t = m_t R_t Q_{t-1} K_t$$
Monetary and Fiscal Policy

Monetary Rule:

\[ i_t = \rho i_{t-1} + (1 - \rho)[\gamma_\pi \pi_t + \gamma_y (y_t - y^n_t)] + \varepsilon_t^{rn} \]

\[ i_t = r_{t+1} - E_t \pi_{t+1} \]

Fiscal Policy:

Gov't spending exogenous and finance by lump sum taxes.
Investment, Finance and Monetary Policy in BGG

\[ I_t/K_t = \phi(Q_t) \]  \hspace{1cm} (6)

\[ E_t R^k_{t+1} = (1 + \chi \left( \frac{Q_t K_{t+1}}{N_{t+1}} \right)) \left\{ (1 + i_t) \frac{P_t}{P_{t+1}} \right\} \]  \hspace{1cm} (7)

where

\[ E_t R^k_{t+1} = E_t \left\{ \frac{P_{w+1}}{P_{t+1}} \alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta)Q_{t+1} \right\} \] \hspace{1cm} (8)
Investment, Finance and Monetary Policy in BGG (con’t)

Note:

\[ N_t = \theta \{(1 - m_t)R_{kt}Q_{t-1}K_t - (1 + i_{t-1}) \frac{P_{t-1}}{P_t} \frac{B_t}{P_{t-1}} \} + (1 - \theta)D \]

Thus:

i. Positive feedback between asset prices and investment (financial accelerator)

ii. Strength depends positively on leverage ratio ratio \( Q_tK_{t+1}/N_t \).

iii. Monetary Policy has additional impact via balance sheets
LOG-LINEARIZED BGG MODEL

Aggregate demand

\[ y_t = \frac{C}{Y} c_t + \frac{I}{Y} \text{inv}_t + \frac{G}{Y} g_t + \frac{C^e}{Y} c^e_t + \ldots \]

\[ c_t = -\sigma r_{t+1} + E_t c_{t+1} \]

\[ c^e_t = \frac{1 - \phi}{\phi} n_{t+1} \]
\[(inv_t - k_t) = \varphi q_t\]

\[E_t r_{kt+1} = (1 - \vartheta) E_t (p_{wt+1} - p_{t+1} + y_{t+1} - k_{t+1}) + \vartheta E_t q_{t+1} - q_t\]

\[E_t r_{kt+1} - r_{t+1} = -v(n_t - q_t - k_{t+1})\]
LOG-LINEARIZED BGG MODEL (con’t)

Aggregate supply

\[ y_t = a_t + \alpha k_t + (1 - \alpha)l_t \]

\[ y_t - l_t = \mu_t + \gamma l_t + c_t \]

\[ \pi_t = \kappa(p_{wt} - p_t) + \beta E_t \pi_{t+1} \]
LOG-LINEARIZED BGG MODEL (con’t)

Evolution of state variables

\[ k_{t+1} = \delta \text{inv}_t + (1 - \delta)k_t \]

\[ n_t = \frac{\theta RK}{N} [r_t^k - r_t] + \theta R(r_t + n_{t-1}) \]

with

\[ r_r = i_{t-1} - \pi_{t-1} \]
LOG-LINEARIZED BGG MODEL (con’t)

Monetary Policy Rule

\[ i_t = \rho i_{t-1} + (1 - \rho)\left[\gamma_\pi \pi_t + \gamma_y (y_t - y^n_t)\right] + \varepsilon^n_t \]

\[ i_t = r_{t+1} - E_t \pi_{t+1} \]
Calibrating Financial Sector Parameters

Choose (i) survival probability $\theta$, (ii) monitoring costs $\mu$, and (iii) the moments of the idiosyncratic shock to match evidence on:

1. Steady state external finance premium: $R_k/R_\ldots$
2. Steady state leverage ration $QK/N$
3. Annual business failure rate.
Figure 3: Monetary Shock - No Investment Delay

Output

Investment

Nominal Interest Rate

Premium

All Panels: Time Horizon in Quarters
Figure 4: Output Response - Alternative Shocks

All Panels: Time Horizon in Quarters
Figure 5: Monetary Shock - One Period Investment Delay

All Panels: Time Horizon in Quarters
Figure 6: Monetary Shock - Multisector Model with Investment Delays

All Panels: Time Horizon in Quarters; Panels 2-4: Model with Financial Accelerator.