ANSWERS TO MIDTERM EXAM

Question 1.

According to the empirical rule 68% of observations are 1 standard deviation away from the mean. This leaves 32% of observations in the two tails. Given that the distribution is symmetric, there will be 16% of observations in each tail.

Therefore, 68+16=84% of observations will be below the mean plus one standard deviation, i.e. below 71+8=79.

Question 2.

(a) From the text of the question we know that:

\[ P(\text{P}) = P(\text{the date is Puertorican}) = 48\% \]
\[ P(\text{R}) = P(\text{the date is Russian}) = 22\% \]
\[ P(\text{I}) = P(\text{the date is Italian}) = 30\% \]

\[ P(\text{C|P}) = P(\text{the date is Catholic given that he is Puertorican}) = 40\% \]
\[ P(\text{C|R}) = P(\text{the date is Catholic given that he is Russian}) = 12\% \]
\[ P(\text{C|I}) = P(\text{the date is Catholic given that he is Italian}) = 80\% \]

Therefore

\[ P(\text{C}) = P(\text{the date is Catholic}) = P(\text{C|P}) \cdot P(\text{P}) + P(\text{C|R}) \cdot P(\text{R}) + P(\text{C|I}) \cdot P(\text{I}) = \]
\[ .40 \cdot .48 + .12 \cdot .22 + .80 \cdot .30 = .46 \]

Applying Bayes’s rule

\[ P(\text{P|C}) = P(\text{C|P}) \cdot P(\text{P}) / P(\text{C}) = .42 \]

(b)

\[ P(\text{I'|C}) = 1 - P(\text{I|C}) = 1 - P(\text{C|I}) \cdot P(\text{I}) / P(\text{C}) = 1 - .52 = .48 \]
Question 3.

(a) The z-score equals
\[ z = \frac{18-30}{4} = -3 \]

(b) The salesman’s claim is likely to be false. According to the empirical rule, if the claim were true, there would be less than 0.15% chances that I buy a batteries that lasts for 18 hours or less.

Question 4.

(a) \( x \), the number of patients admitted in a day, is distributed as a Poisson random variable with \( \lambda = 5.2 \). The probability that the hospital will not have enough beds is:
\[ P(x>8)=1-P(x\leq8) \]

From the table of the Poisson distribution:
\[ P(x\leq8)=0.918 \]

Therefore,
\[ P(x>8)=1-P(x\leq8)= 1 – 0.918 = 0.082 \]

(b) We now that, for a Poisson distribution,
\[ \sigma^2 = \sqrt{\lambda} = \sqrt{5.2} = 2.28 \]

Question 5 (exact solution)

(a) \( P(\text{all three computer are infected}) = \left(\frac{30}{200}\right) \times \left(\frac{29}{199}\right) \times \left(\frac{28}{198}\right) = 0.003 \)

(b) \( P(\text{at least one is infected}) = 1 - P(\text{all non infected}) = 1 – \left(\frac{170}{200}\right) \times \left(\frac{169}{199}\right) \times \left(\frac{168}{198}\right) = 1 – .61 = .39 \)

Question 5 (approximated solution. It assumes that the probability of having an infected computer is the same in all the three draws).

(a) \( P(\text{all three computer are infected}) = \left(\frac{30}{200}\right)^3 = 0.003 \)

(b) \( P(\text{at least one is infected}) = 1 - P(\text{all non infected}) = 1 – \left(\frac{170}{200}\right)^3 = 1 – .61 = .39 \)

Question 6
(a)

\[ P(J) = P(\text{the stock market is up for January}) = .70 \]
\[ P(Y) = P(\text{the stock market is up for the year}) = .80 \]
\[ P(J \land Y) = .63 \]

Therefore, by the multiplicative law,

\[ P(Y|J) = \frac{P(J \land Y)}{P(J)} = \frac{.63}{.70} = .90 \]

(b)

\[ P(Y \land J') = .17 \]

Therefore

\[ P(Y|J') = \frac{P(J \land Y)}{P(J')} = \frac{.17}{.30} = .57 \]

(c)

\[ P(Y) = .80 \]
\[ P(Y|J) = .90 \]

Therefore, being up for January and being up for the year are not independent events.