1 Idiosyncratic Risk and Exogenously Incomplete Markets

We are interested in building a class of models whose equilibria feature a nontrivial distribution of income and wealth across agents. The model is a general equilibrium one: the two building blocks are the "income fluctuation problem" and the aggregate neoclassical production function.

Income fluctuation problem: consider an individual subject to earnings shocks. These shocks are not fully insurable because of the lack of a complete set of Arrow-Debreu contingent claims. We assume that there is only a risk-free asset (i.e. with fixed rate of return) in which the individual can save/borrow, and also that the individual faces an exogenously set borrowing (liquidity) constraint. Individuals who bear a long sequence of bad shocks will have low wealth and will be close to the constraint, individuals with long realizations of good shocks will have high wealth. The process for the labor income shock is exogenous. There are two reasons for savings: intertemporal substitution and precautionary motive, the latter as a self-insurance strategy to hedge against low earnings in the future. Consider a continuum of such agents subject to different shocks: they will give rise to an aggregate supply of capital.

Aggregate production function: profit maximization of the competitive representative firm operating a CRS technology will give rise to an aggregate demand for capital.

General equilibrium: When we let demand and supply interact in an asset market, an equilibrium interest rate will arise. Notice that if a full set of Arrow-Debreu contingent claims were available, the economy would collapse to a representative agent model with a stationary amount of savings such that \((1 + r)^{-1} = 1\). With uninsurable risk, the supply of savings is larger and consequently \((1 + r)^{-1} < 1\).

The questions that can be analyzed in this setup are:

1. How much of the observed wealth inequality can one explain with the existence of uninsurable earnings variation?
2. How much aggregate precautionary savings does the model economy generate?
3. What are the redistributional implications of various policies? How are inequality and welfare affected?
4. Can we generate a reasonable equity premium, once we introduce a risky asset (stock)?

1.1 The Economy: Aiyagari (QJE, 1994)

Demographics: the economy is populated with a measure 1 of ex-ante identical agents, infinitely lived.

Preferences: the individual has time separable preferences

\[
U(c_0; c_1; c_2; \ldots) = E_0 \prod_{t=0}^{\infty} u(c_t)
\]

where the period utility function \(u(c_t)\) satisfies \(u^0 > 0; u^0 < 0\) and the discount factor \(\beta < 1\). The expectation is over future sequences of shocks, conditional to the realization at time 0. The individual supplies labor inelastically.

Endowment: each individual has a stochastic endowment of efficiency units of labor \(f; 2 \infty \ldots \ldots \infty \). The shocks follow a Markov process with transition probabilities \(\gamma_{t+1}^{i} = \gamma_{t}^{j} \) and they are iid across individuals. We assume a law of large numbers to hold, so that \(\gamma_{t}^{i} \) is also the fraction of agents in the population subject to this particular transition.
2 Budget constraint:
\[ c + a^0 = Ra + w^0 \]
where \( c \) is current consumption, \( a^0 \) is next period wealth, \( R = (1 + r) \) is the gross interest rate and \( w \) is the wage rate.

2 Liquidity constraint:
\[ a^0 \geq b \]
where \( b \) is exogenously specified ("ad-hoc borrowing constraint"). Is there a "natural" borrowing constraint, i.e. the maximum amount which is always repayable by the household no matter how unlucky she is? Impose \( a \geq 0 \) at every \( t \) and iterate forward over the budget constraint assuming the worst possible realization of the shocks, i.e. "forever, then
\[ a \geq 0 \]

which the present value of the lowest possible realization of her future earnings. This condition is also called "almost sure present value budget balance". We conclude by defining, as borrowing constraint,
\[ a^0 \geq \frac{\min \{ \frac{n w^0}{r} \}}{1 + r} \]

2 Technology: assume a CRS production function \( F(K; H) \) operated by a competitive firm

2 Market structure: final good market (consumption and investment goods), labor market, and capital market are competitive.

2 Aggregate resource constraint (aggregate feasibility condition):
\[ F(K; H) = C + I = C + K^0 \]

1.2 Definition of Stationary Equilibrium

We analyze the steady-state of the economy where all the aggregate variables are constant (but individuals move up and down in the earnings and wealth distribution!).

At each point in time, the individual is characterized by the pair \((a;\alpha)\) -the individual states. The aggregate state of the economy is the distribution of agents across states, i.e. \(\pi(a;\alpha)\). We would like this object to be a measure, so we need to define an appropriate mathematical structure. Let \(\mathcal{A}\) be the maximum asset holding in the economy. Define the set \[ A = \left\{ a \left| a \in \mathbb{R} \right. \right\} \]
and the set \(B = \mathbb{R}\). The space \((S; B(S))\) is a measurable space, and for any \(S \leq S\), \(\pi(S)\) is the measure of agents in the set \(S\).

How can we characterize the way individuals transit across states over time? I.e. how do we obtain next period distribution, given this period distribution? We need a transition function. Define \(Q((a;\alpha); S)\) as the probability that an individual with state \((a;\alpha)\) next period belongs to the set \(S\), formally \(Q: S \leq B(S) \rightarrow [0;1]\), and
\[ Q((a;\alpha); S) = \sum_{j \in B(S)} Q((a;\alpha); j) \]

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where \( I \) is the indicator function, and \( a^0 = g_a(a;") \) is the optimal saving policy. Then \( Q \) is our transition function, because

\[
Z_1(A \in E) = \int_{A \in \mathbb{E}} Q((a;");A \in E) d\mu_0(a;"
\]

Let us now re-state the problem of the individual in recursive form, i.e. through dynamic programming

\[
v(a;";\mu) = \max_{c; a^0} \left( u(c) + \sum_{a^0;`\mu} v(a^0;";\mu) \right)
\]

s.t:

\[
c + a^0 = R(t) a + w(t)"
\]

where we have made explicit the dependence of prices from the distribution of agents (although it is redundant in a stationary environment).

A stationary recursive competitive equilibrium is a value function \( v : S \rightarrow \mathbb{R} \), policy functions \( g_a : S \rightarrow \mathbb{R} \), and \( g_c : S \rightarrow \mathbb{R} \), prices \( R \) and \( w \) and a stationary measure \( \mu \) such that:

1. given prices \( R \) and \( w \), the policy functions \( g_a \) and \( g_c \) solve the household's Bellman equation and \( v \) is the associated value function.

2. given \( R \), the rm chooses optimally its capital \( K \) and its labor \( H \), i.e. \( r + \pm = F(K;H) \) and \( w = F_H(K;H) \)

3. the labor market clears: \( H = R(A \in \mathbb{E}) g_a(a;";\mu) d\mu(a;"
\]

4. the asset market clears: \( K = R(A \in \mathbb{E}) g_a(a;";\mu) d\mu(a;"
\]

5. the goods market clears: \( R(A \in \mathbb{E}) g_c(a;";\mu) d\mu(a;"
\]

6. for all \( A \in \mathbb{E} \in B(S) \), the stationary measure \( \mu \) satisfies

\[
Z_\mu(A \in \mathbb{E}) = \int_{A \in \mathbb{E}} Q((a;");A \in \mathbb{E}) d\mu(a;"
\]

where \( Q \) is the transition function defined above.

Remark: a stationary distribution \( \mu \) implies that the cross-sectional distribution is constant over time, while individual asset holdings vary stochastically and individuals move around within the distribution.

1.3 An Algorithm for the Computation of the Equilibrium

How do we compute this equilibrium? The algorithm that can be used is a fixed point algorithm over the interest rate.

1. Fix an initial guess for the interest rate \( r^0 \in \frac{1}{2} \pm \frac{1}{2} \in \frac{1}{2} \) : Later we explain why an equilibrium interest rate will always belong to that set. Given the interest rate, obtain the wage rate \( w(r^0) \) using the CRS property and solve the dynamic programming problem of the agent to obtain \( g_a(a;";r^0) \).
2. Given $g_t(a; r^0)$ and $y^0$, we can construct the transition function $Q^i r^0 \xi$ and through an iterative procedure, obtain $1^i r^0$

3. Compute $H$ from the labor market clearing condition, and the aggregate demand of capital $K^i r^0 \xi$ from the optimal choice of the firm who takes as given $r^0$.

4. Compute the integral
$$A^i r^0 \xi = \int_Z g_t(a; r^0)\,dE(a; r^0)$$

which gives the aggregate supply assets.

5. Compare $K^i r^0 \xi$ with $A^i r^0 \xi$. If $K^i r^0 \xi < (>) A^i r^0 \xi$ then the next guess $r^1 < (>) r^0$: To obtain $r^1$ a good choice is
$$r^1 = \frac{1}{2} r^0 + \int F(K^i r^0 \xi; H) \, d\xi$$

6. Update your guess to $r^1$ and keep iterating until convergence of the interest rate to $r^\infty$.

1.4 Properties of the Stationary Equilibrium

1.4.1 Existence of an upper bound for the set $A$

First we show a necessary condition for the existence of an upper bound $\bar{a}$ in the asset space $A$: Consider the Euler equation implied by the income fluctuation problem:
$$u^q_t = (1 + r)E_t u^q_t(b_{t+1})$$

and define $M_t = (1 + r)^t u^q_t(c_t)$. Then it follows that
$$E_t(M_{t+1}) = (1 + r)E_t[(1 + r)E_t u^q_t(b_{t+1})] = (1 + r)^t u^q_t(c_t) = M_t$$

hence $M_t$ is a supermartingale. The convergence theorem for supermartingales says that $M_t \uparrow \bar{a}$ finitely. We need to examine 3 cases:

1. $0 < (1 + r) < 1$, then convergence implies $u^q_t(c_t) \uparrow 0$ and therefore $c_t \uparrow 0$. Given the borrowing constraint, also $a_t$ diverges.

2. $0 \geq (1 + r) > 1$. Assume for simplicity that $\bar{a} = 0$: We proceed by contradiction: we will first suppose that convergence of the supermartingale implies that there exists an upper bound for $a_t$ called $\bar{a}$. Then, we will show the contradiction. Consider an individual who has had a long winning streak of $1$ and has reached $\bar{a}$. From the Euler equation and the envelope condition
$$v^q(b; y), (1 + r)E_t v^q(b; y) > (1 + r) v^q(b; y) = (1 + r) v^q(b; y) = v^q(b; y)$$

which leads to a contradiction, hence $a_t$ is unbounded. Intuitively, for $(1 + r) > 1$ the individuals want to smooth consumption perfectly. Keeping a fixed amount of assets implies consuming proportionally to its endowment which leads to a very volatile consumption path that, given risk-aversion, is not optimal.

3. $0 < (1 + r) < 1$, then convergence could occur for a finite value $\bar{a}$.

Let us now learn more about the uniqueness of the stationary distribution. Define a closed order $\s^0$ on the set $\s$ with generic element $s = (a; r^0)$. For any pair $(s; s^0) \in \s$
$$s \s^0 \text{ iff } (a, a^0 \text{ and } r^0, r^0) \text{ or } (s^0 = (a; r^0)) \text{ or } (s = (a; r^0))$$
1.4.2 Uniqueness of the Stationary Distribution

Let us state a useful property of $Q$, called

Monotone Mixing Condition (MMC): there exists $s^a \geq 2$ and $\varepsilon > 0$ and $N$ such that

$$Q^N((a; s; g); fs : s < s^a g) > \varepsilon \quad \text{and} \quad Q^N((a; s; g); fs : s > s^a g) > \varepsilon$$

i.e. the probability of moving away from the worst state $(a; s; g)$ in a finite number of periods $(N)$ is positive (the "American dream"), and so is the probability of moving enough far away from the best state in a finite number of periods (the "American nightmare").

We can now determine conditions for the uniqueness of the stationary distribution, which is a necessary condition for uniqueness of the equilibrium.

Theorem 1 (Hopenhayn and Prescott, 1987): If $S$ is a compact metric space, $\pi$ is a closed order on $S$, $S \subseteq B(S)$ is a measurable space, $Q$ is a transition function on $S \subseteq B(S)$, $Q$ is increasing and the MMC condition is satisfied; then, there exists a unique stationary distribution $\pi$ which is found applying iteratively the transition equation (1)

But can we use this theorem in our economy? First, we have shown above that when $\bar{r}(1 + r) < 1$, then $S$ is compact. Moreover, the order $\pi$ defined above is closed. Proving the monotonicity of $Q$ is more difficult, but Huggett (1993) shows that if $g(a; s; r)$ is increasing in $a$ for given $s; r$, then this is verified. To show that the MMC holds, given the monotonicity of $g$, just start from the worst (best) state and construct a long enough realization of the best (worst) shocks, which is an event with positive probability) such that the asset holdings cross a specified threshold $a^a$.

1.4.3 Supply and Demand for Capital

Consider now the aggregate demand of capital. From the optimal choice of the firm, we can obtain

$$K(r) = F_k^{-1}(r + \lambda H(\pi(r)))$$

Note that for $r = \bar{r}$, then $K(r) = +1$, while for $r = \lambda$, $K(r) = 0$. Moreover, $K(\pi(r)) < 0$:

Consider the aggregate supply of capital

$$A(r) = \int_{a \in E} g_a(a; s; r) d\pi(a; s; r)$$

Suppose first $(1 + r) < 1$, i.e. $r = \frac{1}{2}$, then as we showed above, the aggregate supply of assets goes to infinity, i.e. $A(\frac{1}{2}; 1) = +1$. The intuition is simple: individuals like savings for precautionary reasons, however, if $\bar{r} < 1 = (1 + r)$ (i.e. $1 > r$), then there is a cost to postpone consumption and aggregate savings could be bounded above. When $\frac{1}{2} > r$, this cost is zero, so households accumulate assets in, not because any finite amount of precautionary savings will be exhausted in presence of a long enough bad realization of the shocks. For $r = \bar{r}$ the individual would like to borrow until the limit as she never repays, so $A(\bar{r}; 1) = \bar{b}$ as the natural borrowing limit would be positive with $r = \bar{r}; 1$; thus $A = A$.

Existence of the equilibrium: we need continuity of the functions $K(r)$ and $A(r)$. Assuming continuity of $K$ on $r$, and continuity of $g_a$ in $r$, also $A(r)$ and $K(r)$ are continuous.

Uniqueness of the equilibrium: we need uniqueness of the stationary distribution. Given that $K(\pi(r)) < 0$, then we only need to ensure that $A(\pi(r)) > 0$. 

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1.5 A Graphical View of the Equilibrium

1.5.1 Precautionary savings

The graph also allows to examine the amount of savings that would occur in the economy with full insurance. Under full insurance, the Euler equation implies \((1 + r)\bar{F} = 1\), hence the supply of capital is infinitely elastic at \(r = 1\). The point where this horizontal line crosses \(K(r)\), represents the savings of the neoclassical deterministic growth model, call them \(K^F\). The magnitude \(\bar{F} = K^F\) is the amount of aggregate precautionary savings.

Some remarks are in order. First, in partial equilibrium, i.e. with \(r\) arbitrarily set, one can generate any amount of precautionary savings. Second, the “conventional wisdom” about the convexity of marginal utility, i.e. \(u^{\infty} > 0\) is plain wrong in general equilibrium. Infact, in general equilibrium, one does not need that condition to generate precautionary savings:

Theorem 2 (Huggett and Ospina, 2000): precautionary savings arise in equilibrium if individuals (1) are risk averse, and (2) face borrowing constraints that bind in equilibrium with positive probability (i.e. for a positive measure of agents).

1.5.2 Comparative Statics

Suppose we increase \(b\), i.e. we slacken the liquidity constraint and increase the maximum amount that can be borrowed by the individual. Graphically, the asset supply curve \(A(r)\) shifts upward, with \(K(r)\) constant, which leads to a rise in the interest rate, a reduction of precautionary savings and a rise in wealth inequality. The interest rate increases because more individuals have negative wealth, so the supply of capital falls at any given \(r\). Precautionary savings are also reduced because agents do not need to worry as much to hedge against bad times, since they can borrow more. Finally, a slacker borrowing constraint means that there will be more poor and unlucky individuals holding (more) negative wealth, while the income-rich individuals do barely change their behaviour. So wealth inequality is greater because of larger dispersion in the bottom of the distribution.

Suppose we increase \(z\) of the aggregate production function \(F(z; K; h)\). Both curves shift to the right: the \(K(r)\) curve shifts because higher productivity of capital increases its demand. The \(A(r)\) curve shifts because the increase in productivity raises the labor income \(w\) of the agents who consume and save more. The change in \(A\) has no effect either on the equilibrium interest rate or on the fraction of precautionary savings or on wealth inequality. Homework: how do we need to specify the borrowing constraint (as a function of \(r; A(r)\)) to maintain this neutrality result?

Suppose we increase the variance of the uninsurable income shock. \(K(r)\) is unchanged, but the supply of capital \(A(r)\) would go up, as individuals cumulate more savings to cope with the higher uninsurable uncertainty of their income. Wealth inequality would rise, as more dispersed income would translate (less than proportionally) into more dispersed wealth. Increasing the persistence of the shock has a very similar effect.
1.6 Matching Wealth Inequality

What are the facts that the model aims at reproducing? For a full description of facts on earnings and wealth inequality (and more), you should refer to a recent paper by Budria, Diaz-Gimenez, Quadrini and Rios-Rull) available at http://www.ssc.upenn.edu/~vr0j/papers/qr2j01.pdf. The following table, reproduced from the paper above, provides the key statistics for the US (from the Survey of Consumer Finances, 1998)

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Gini</th>
<th>CV</th>
<th>Q1</th>
<th>Q3</th>
<th>Q5</th>
<th>Top 1%</th>
<th>Top 5%</th>
<th>Bottom 5%</th>
<th>Share of top 1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings</td>
<td>21.1</td>
<td>1.57</td>
<td>.61</td>
<td>2.65</td>
<td>-7</td>
<td>20.7</td>
<td>101.9</td>
<td>491</td>
<td>136</td>
<td>0.0</td>
<td>7.5%</td>
</tr>
<tr>
<td>Wealth</td>
<td>47.4</td>
<td>4.03</td>
<td>.80</td>
<td>6.53</td>
<td>-2.3</td>
<td>51</td>
<td>770</td>
<td>5,988</td>
<td>1,150</td>
<td>-4.7</td>
<td>31%</td>
</tr>
</tbody>
</table>

There are 4 key facts to observe: 1) wealth is much more unequally distributed than earnings, 2) both distributions are very skewed (mean>median), but in particular the wealth distribution, and 3) the top 1% of the wealth distribution owns 30% of the US wealth, 4) persistence in the top is very high, but particularly so for earnings.

Once solved, the model needs to be calibrated. A standard calibration would be as follows. With CES utility function, choose $\gamma = 2$ for the risk aversion parameter. Set $\bar{\sigma}$ to match the aggregate capital/output ratio in the model, with annual data this ratio equals 3. With Cobb-Douglas technology, set the capital share $\bar{\sigma} = .3$. Normalize the productivity parameter $z$ to reproduce an aggregate output of, say, 1. Set the depreciation rate to $\delta = .06$. Calibrating a value for the borrowing constraint $b$ is more difficult, as there is no direct information on the fraction of agents who are constrained in the economy. It is important to do some sensitivity analysis on this parameter.

The key parameters to calibrate are those of the exogenous earnings process. We need panel data on earnings $e_{it}$ for individuals $i = 1; \ldots; I$ at time $t = 1; \ldots; T$. A typical econometric model for earnings would be

$$e_{it} = a_i + g(x_{it}) + \xi_{it}$$

(3)

where $a_i$ is the individual fixed effect, $g(x_{it})$ is a nonlinear function of observable variable that determines income (experience, education, gender, race, etc...), while $\xi_{it}$ is the unpredictable part. Notice that it is only $\xi_{it}$ that is relevant for our model. Usually $\xi_{it}$ is modelled as

$$\xi_{it} = z_{it} + \eta_{it}$$

$$z_{it} = \frac{1}{2} z_{i,t-1} + \gamma_{it}$$

where $\eta_{it}$ is an innovation of a transitory nature, while $\gamma_{it}$ is an innovation of a permanent nature, as $z_{it}$ is the component that displays persistence, given $\gamma > 0$: Usually, the estimates imply $\gamma = 2$ [$2; 4$], while $\gamma = .6$. However, some authors criticize the specification (3) with $\gamma$ fixed and argue that the latter are very difficult to estimate consistently as a result of the rather short time-dimension of the panel data available. For this reason they omit $a_i$ from (3) and as a consequence they find much more persistent shocks, i.e. $\gamma = .9$. The chosen value for $\gamma$ can change substantially the results. For various approaches to this problem, see Heaton and Lucas (JPE 1996), Storesletten, Telmer and Yaron (various forthcoming), Krussell and Smith (JPE, 1998).

For computational reasons, it is useful to “discretize” the state space for the shock in to a finite grid as defined above and an associated Markov chain. How do approximate an AR(1) into a Markov chain? See Tauchen (Economic Letters, 1986). Notice that with a $n$ point Markov chain, we have $(n+1)^2$ parameters to choose, i.e. $n+1$ points of the grid -if we normalize say the highest value in order to get average earnings equal to 1- and $n(n+1)$ transition probabilities -given that probabilities have to sum to 1 along each row of the Markov chain.
Next the model is simulated, the equilibrium is computed and we can ask the model the following question: how far can we go in explaining wealth inequality when idiosyncratic earnings shocks are the only source of heterogeneity? The answer is that the standard model does well in matching the wealth inequality in the bottom half of the distribution, while it fails to match the fact that very few individuals own so much of the aggregate wealth. In particular, the Gini generated by the model economy is around 0.4 much smaller that the data value 0.8. One reason for the distance between model and data is that in the model there are no ex-ante differences across households, such as age and education that would introduce further wealth inequality.

Remember that one can use the model as a “measurement tool” to assess the amount of aggregate precautionary savings in the economy. The results depend a lot on parametrization. With log utility and $\frac{1}{2} = 0$, the answer is approximately zero (probably a lower bound). With $\frac{1}{2} = 5$ and $\frac{1}{2} = 0.9$ the answer is 14% (probably an upper bound). For $\frac{1}{2} = 2$ and $\frac{1}{2} = 0.8$, we find a precautionary saving rate of roughly 2%, still quite small.

2 The Role of Public Insurance

In an economy where some of the earnings risk is uninsurable because of market incompleteness, there could be scope for public insurance, i.e. government intervention through taxation and redistribution from the rich-lucky to the poor-unlucky. If the government could implement lump-sum taxation, then the public program could provide an effective margin for consumption smoothing without too much loss in terms of efficiency. Unfortunately, lump-sum taxation is more of a theoretical benchmark than a practical possibility and taxation is distortionary. Therefore, there is a trade-off between insurance and efficiency.

We develop now a variation of the Aiyagari economy where the government is an additional agent of the economy and where one can evaluate such trade-offs. The model follows Floden and Linde (RED, 2001). We need to introduce a number of modifications to the benchmark economy.

Preferences
\[ U(c_0; c_1; \ldots; l_1; l_2; \ldots) = \sum_{t=0}^{\infty} e^{-\frac{t}{\beta}} u(c_t; l_t) \]
i.e. we introduce leisure $l_t \in [0; 1]$ in order to have a margin where distortions matter. This means that we will have an optimal policy for labor supply $h = 1 - l = g_t(a; \eta)$. Notice also that the agents might be using their elastic labor supply to self-insure. Take an agent who is liquidity constrained and has a low realization of the productivity shock: to keep his consumption high, he could intensify his labor supply. Finally, with leisure the equilibrium condition in the labor market becomes
\[ H = \int_{\mathbb{A}} g_t(a; \eta) d\eta g_t(a; \eta) \]

Budget constraint:
\[ c + a^0 = R a + (1 - \xi) w (1 - l) + b \]
where $\xi$ is a flat earnings tax and $b$ is the lump-sum transfer of the government. Clearly it would be more efficient to condition the transfer on $\eta$ (i.e. agent with low $\eta$ would receive more), but Floden-Linde assume that $\eta$ is unobservable to the government.

Government budget constraint (balanced in equilibrium)
\[ b = \xi w \int_{\mathbb{A}} g_t(a; \eta) d\eta g_t(a; \eta) \]
The definition of the recursive competitive equilibrium for this economy is very similar to the benchmark case (the chief differences are the existence of a decision rule for leisure and of the balanced budget condition of the government).

2.1 Results

Floden-Linde calibrate this economy without taxes to the US, i.e. estimate the earnings shock on US data ($\sigma_{US} = .9$) and they ask the following question: what is the level of government redistribution that maximizes the “average” utility of individuals in the economy? What welfare gains does such redistribution imply for individuals?

In an economy with heterogeneous agents there is not a unique welfare function, it all depends on what weights are assigned to each type. Floden-Linde assume an equal weight welfare function, i.e. they solve

$$\max_{\zeta} W(\zeta) = \int u(g_c(a;'; \zeta); 1; g_t(a;'; \zeta)) d\Phi(a;'; \zeta);$$

with the constraint that the allocations are a competitive equilibrium. Intuitively, for low levels of redistribution, welfare is low because individuals have a large amount of undesired consumption fluctuations; for very high levels of taxes insurance is very good but at the same time heavy distortions on labor supply are imposed. So, there will be an interior level of $\zeta$, call it $\zeta^*$, that maximizes welfare.

For the U.S., they find that $\zeta^*_{US} = .27$ and with this much of redistribution the individual, on average, can increase her yearly consumption by 5-6% per year, compared to the case where $\zeta = 0$.

So welfare gains from public insurance programs in the US are quite large. Does it mean that public insurance is always good? Floden and Linde examine the case of Sweden, a country that traditionally has heavy government intervention and generous welfare programs. They calibrate the model to Sweden. The major difference is the earnings process: shocks are much less variable and less persistent than the U.S. ($\sigma_{Sweden} = .8$), so earnings fluctuations in Sweden are much more insurable. It shouldn’t come as a surprise then that they find an “optimal” tax rate $\zeta^*_{Sweden} = .03$, i.e. very low. Essentially, earnings fluctuations in Sweden are small and individuals can largely self-insure. Clearly, the amount of actual government transfers in Sweden is much larger than 3%, so in this sense there is “too much” public insurance in Sweden, as the optimum amount is exceeded the tax-induced distortions can be quite costly.

Finally a note: the authors only consider a flat earnings tax. Often governments use capital income tax for redistribution which is much more distortionary. Governments also use progressive taxes: one good feature of this type of taxation is that it reduces the variability of earnings (by taxing proportionately more high earnings and less low earnings), so this margin could represent an additional source of insurance.

**HOMEWORK:** consider the Floden-Linde economy and add government debt $D_t$. Describe the new economy, define the equilibrium, draw the asset market equilibrium graph and try to disentangle all the possible effects of government debt. Is there any sense in which debt alleviates the liquidity constraints of the agents?
\[ \frac{1}{\beta} - 1 - \delta \]

\[ \Lambda(r) \]

\[ K(r) \]

\[ r \]

\[ -b \]

\[ -\delta \]

\[ -1 \]