1 Economies with Limited Enforcement

This class of models allows to describe economies where the “market incompleteness” that prevents full consumption insurance is endogenous. The starting point is the Arrow-Debreu economy with a complete set of insurance markets available in principle. In the Arrow-Debreu economy though there is an implicit key assumption made: all contracts traded at time zero are then fully enforceable in the future. We introduce now an enforcement friction in the Arrow-Debreu world: a contract signed in the past between two agents will be carried out only if both agents have an incentive to do so in the present: one of the two agents might ...nd more pro...table to default that to participate in the contract.

In light of this event, some contracts will never be signed in equilibrium and therefore the equilibrium will display some missing markets and only partial consumption insurance. In particular, agents will lend up to the point where borrowers will have the incentive to repay in the future, so although the key element leading to market incompleteness is the threat of default, there will never be default in equilibrium.

1.1 The Economy: Kocherlakota (REStud, 1996)

Demographics: The economy is populated by two types of in...nitely lived households indexed by \( i = 1; 2 \).

Endowment: Let \( y_i^t \) denote the stochastic endowment to agent \( i \) at time \( t \), where \( y_i^t \in Y \) \( \in \{ y_1; y_2; \ldots; y_s; \ldots; y_S \} \) g. i.e. it is a ..nite set with cardinality \( S \). The stochastic process governing the shock is iid over time with probabilities \( \frac{1}{S} \) \( \cdot Pr(y_i^t = y_s) \). We assume that the aggregate endowment is stationary and normalized to one, i.e. \( y_1^t + \cdots + y_s^t = 1 \). We also assume that the endowment process is symmetric, i.e. \( Pr(y_1^t = y_s) = Pr(y_s^t = y_1) \). Example: \( Y = \{ 0; 1; 2; \ldots; 9 \} \), with \( \frac{1}{10} \) \( \cdot \) \( \sum_{s=1}^{10} y_s^t = 1 \).

We de...ne a history for the economy at time \( t \) as a sequence of realizations of endowments for type \( 1 \) from time 0 to time \( t \):

\[
h_t = (y_1^0; y_1^1; \ldots; y_1^t; \ldots; y_s^t) \]

The realizations for type \( 2 \) are implied by the stationarity of the aggregate endowment. The variable \( h_t \) is the aggregate state of the economy, as it summarizes all the relevant information, i.e. everything that happened in the economy up to that point. Call \( H_t \) the set of all possible histories up to time \( t \).

Preferences: Preferences for type \( i \) are de...ned over consumption streams \( c_i^t \) \( \cdot \) \( c_i^t(h_t) \) \( \cdot \) \( t = 0 \):

\[
V(c_i^t) = \sum_{t=0}^{\infty} \bar{\gamma}^t u(c_i^t) = \sum_{t=0}^{\infty} \bar{\gamma}^t u(c_i^t(h_t)) \cdot 2H_t, \]

where the discount factor \( \bar{\gamma} \) \( \in (0; 1) \) and \( u \) is strictly increasing, strictly concave, \( C^2 \) and satis...es the Inada conditions.

Market Arrangements: In this economy there are strong incentives to share the risk stemming from random income fluctuations. However, agents cannot commit to long-term contracts, so they face an “individual rationality constraint” stating that the value of participating in the risk-sharing agreements must be superior to the alternative, i.e. defaulting. Suppose that this alternative is the worst possible punishment from having defaulted on the contract, i.e. exclusion from ...nancial markets
thereon. At time $t$, the defaulting agent will consume $u(y_i)$ and he will be leaving in autarky from there on consuming his own stochastic endowment forever, with associated continuation utility

$$V^\text{aut} = \sum_{j=1}^{H_t} u(y_{h+t,j}^{i+1})$$  \hspace{1cm} (1)

The implied individual rationality constraint then must say that the allocations for type $i$ implied by the contract should offer at least the value of defaulting to each agent at each particular history $h_t$, i.e.

$$u(c_i^{h_t}) + \sum_{j=1}^{H_t} u(\bar{y}_{h+t,j}^{i+1}) u_i y_i + V^\text{aut}.$$  \hspace{1cm} (2)

Instead of defining an equilibrium for this economy, we start from the efficient and constrained efficient allocations and next we explain how one can decentralize such allocations. Establishing the equivalence gives us also a simpler way to compute allocations, using a Planner’s problem (i.e. without worrying about prices and market clearing conditions).

**1.2 Efficient and Constrained Efficient Allocations**

**Definition 1** The (unconstrained) Pareto optimal allocations solve the problem

$$V(\psi) = \max_{c_1, c_2} V(c_1, c_2)$$  \hspace{1cm} (3)

s.t:

$$c_1 + c_2 = y_1 + y_2$$

where the function $V(\psi)$ describes the Pareto frontier and represents the maximum utility that agent 2 can obtain from any feasible allocation (feasibility constraint), once agent 1 is promised at least $\psi$ (promise-keeping constraint).

Homework: show that a Pareto Optimal allocation implies $u(c_1^{h_t}) = u(c_2^{h_t})$ constant for any $h_t$, and prove that splitting the aggregate endowment exactly in half forever is an optimal allocation.

We are interested in characterizing the constrained-Pareto optimal allocations, i.e. the allocations that solve the problem (3) subject to the additional individual rationality constraints (3) $V^\text{v} c_i^h \geq V^\text{aut}$.

These extra constraints capture the absence of full enforcement: the two agents in the economy give up their own endowments to the planner and agree to let the planner choose how to share the aggregate endowment between the two, but since the planner does not have a technology to enforce perfectly this agreement, she has to keep the two agents in the contract through incentives, and this is the role of the extra constraints.

The aggregate state of the above economy is the full history of realizations of the endowment shocks. As $t \to 1$, the dimensionality of the state blows up and any computation becomes unfeasible. Fortunately, there exists a recursive representation of the Pareto problem.

**1.2.1 Recursive representation of the Pareto problem**

The key step is to use, as state variable, the promised future utility to one of the individuals, in the same way as $\psi$ is a state of the Pareto frontier problem (3). Denote therefore the state for the planner
as $v$, which represents the future utility promised to agent 1 one period earlier. As in Kocherlakota (1996), we adopt the timing convention that the Planner chooses, before the realization of the state of the world $y_s$, the pair of consumption for agent 1 and promised future utility to agent 1 ($c_s; v_s$) contingent on the realization of the shock for agent 1, $y_s$. Then, after the realization, each individual can decide to honour the contract or quit. Notice that, for the sake of simplifying the notation, we have used the convention that $c_s$ and $y_s$ refer to individual 1.

Then the recursive formulation of the constrained-efficient problem is:

$V(\psi) = \max_{(c_s; v_s) \in S} \left\{ X [u(c_s) + v_s]^{1/4} \right\}$

subject to:

- $u(c_s) + v_s \geq y_s$, for all $s$ (PK)
- $u(c_s) + v_s \geq u(y_s) + v_{aut}$, for all $s$ (IR1)
- $u(c_s) + v_s \geq u(y_s) + v_{aut}$, for all $s$ (IR2)

where we have used the feasibility constraint $1 = c_1 + c_2$, for all $s$. Inequality (PK) is the promise-keeping constraint ensuring that the utility promised last period is actually delivered; (IR1) is the individual rationality constraint for agent 1 and (IR2) is the individual rationality constraint for agent 2 which in turn ensure that the agents always participate into the contract, no matter what shock is realized, i.e. the contract is sustainable. The last constraint says that, upon realization of the shock $y_s$, promised future utility $v_s$ must belong to the compact utility space bounded between $V_{aut}$ and $V_{max}$.

Note that the Planner decides how much consumption $c_s$ to give to agent 1 and how much utility $v_s$ to promise for the future, contingent on the realization of $y_s$. In doing so she must take into account what she promised the previous period and the fact that if the agent receives too little current consumption, he might choose not to participate in the risk-sharing agreement.

To construct a solution of the problem above, the first step is to solve for the value of autarky $V_{aut}$ and the maximal utility $V_{max}$. Computing the value of autarky is extremely simple given the iid nature of the shock:

$$V_{aut} = \frac{1}{4} \sum_{s=1}^{X} u(y_{s})^{1/4};$$

Homework: suppose the shocks are Markov with transition probabilities $\frac{1}{4}$($s$)($s$). Define the values of autarky (which are now state dependent) and suggest a way to solve for them, independently of the Planner’s problem.

Computing the state contingent maximal value $V_{max}$ requires the knowledge of the value function $V$: From the objective function of the Planner and the (IR2), it follows that:

$$V(\psi) = \max_{s \in S} \left\{ X [u(1; y_s) + v_{aut}]^{1/4} \right\};$$

and since utility to agent 2 $V$ declines in the promised utility for agent 1, $V_{max}$ solves

$$V(V_{max}) = \max_{s \in S} \left\{ X [u(1; y_s) + v_{aut}]^{1/4} \right\};$$

thus $V_{max}$ is the maximum utility value that can be promised to agent 1 without violating the fact that agent 2 can always opt for autarky at any time. Later, we explain how to compute $V$. 

3
1.3 Characterization of the Solution

The (recursive) Lagrangian associated to the constrained Planner’s problem above is

\[ L = \sum_{s \in S} X \left[ u(1_i, c_s) + \hat{\nu} (v_s) \right] \frac{1}{s} + \sum_{s \in S^2} \mathbf{1}^F \left[ u(c_s) + \hat{\nu} (v_s) \right] i - \mathbf{V}^{\text{aut}} \frac{V}{V^{\text{aut}}} \]

The associated first-order conditions with respect to \( c_s \) is:

\[ i u(1_i, c_s) \frac{1}{s} + \sum_{s \in S^2} \mathbf{1}^F \left[ u(c_s) + \hat{\nu} (v_s) \right] i = 0 \quad (5) \]

where the interior solution is guaranteed by the strict concavity of \( u \). The associated first-order condition with respect to \( v_s \) is:

\[ \mathbf{V}^{\text{aut}} \frac{V}{V^{\text{aut}}} \left[ u(1_i, c_s) \right] \frac{1}{s} + \sum_{s \in S^2} \mathbf{1}^F \left[ u(c_s) + \hat{\nu} (v_s) \right] i = 0 \quad (6) \]

Equations (5) and (6) imply the following relation between \( c_s \) and \( v_s \):

\[ \frac{u(1_i, c_s)}{u(c_s)} = \frac{\mathbf{V}^{\text{aut}} \frac{V}{V^{\text{aut}}} (v_s)}{\mathbf{V}^{\text{aut}} \frac{V}{V^{\text{aut}}} (v_s)} = \frac{u(1_i, c_s)}{u(c_s)} \quad (7) \]

which equalizes the marginal rates of substitutions between \( c_s \) and \( v_s \) (i.e. the ratio of marginal utilities) across agents. Moreover, from the Envelope Theorem

\[ L \left( v \right) = \mathbf{V}^{\text{aut}} \frac{V}{V^{\text{aut}}} = i \frac{1}{i}; \quad (8) \]

i.e. the multiplier \( i \) measures how much the value of the contract declines for agent 2, as the utility promised to agent 1 increases.

Now, we are ready to show that we can be in one of three possible situations, according to the value of \( y_s \): 1) states where (IR1) binds, 2) states where (IR2) binds, 3) states where both constraints are slack. In particular, we cannot have a case where they are both binding.

Lemma 2 (IR1) and (IR2) cannot be both binding at the same time.

We provide a simple proof by contradiction. When (IR1) is binding, \( u(c_s) + \hat{\nu} (v_s) = u(y_s) + \mathbf{V}^{\text{aut}} \), thus \( c_s \cdot y_s \) since \( v_s \cdot V^{\text{aut}} \): When (IR2) is binding, for the same reasoning, \( u(1_i, c_s) \cdot u(1_i, y_s) \), thus \( c_s \cdot y_s \). Then it must be that \( c_s = y_s \) and therefore \( v_s = V^{\text{aut}} \) and \( V (V^{\text{aut}}) = V^{\text{aut}} \). For this latter equality to be true, the autarkic allocations would need to be the only constrained Pareto efficient allocations, which is not true unless the Pareto frontier is degenerate. In general, \( V (V^{\text{aut}}) > V^{\text{aut}} \). The three cases are described below:

1. Consider first the region where none of the (IR) constraints bind, i.e. \( i \frac{1}{s} = i \frac{2}{s} = 0 \). From the two first-order conditions and the envelope condition, we obtain

\[ \frac{1}{s} u(1_i, c_s) = u(1_i, c_s) \quad V^{\text{aut}} (v_s) = i \frac{1}{i} = V^{\text{aut}} (v_s) \]

which imply that promised utility does not change over time, i.e. \( v_s \) is constant and so is \( c_s \), hence there is full risk-sharing as consumption is completely isolated from income fluctuations. In this
2. Consider now the region where (IR1) binds, i.e. the pair agent 2 who, in turn, has a fall in consumption, but not as big as in autarky. In this situation, but by promising him also higher future consumption, she induces agent 1 to insure partially and the envelope condition, we obtain conclusion, when agent high and autarky becomes attractive for type 1. From the ...rst order condition with respect to \( c_s \), we obtain

\[
V^q(v_s) = V^q(u) \frac{1}{\frac{1}{v_s} + \frac{1}{s^0}}
\]

from which it follows that \( v_s > v \) since the value function is concave and \( V^q(v_s) < V^q(u) \). Hence, when the (IR1) is binding the promised utility to agent 1 is increased to keep him away from the participation constraint. What happens to \( c_s \)? From the ...rst order condition with respect to \( c_s \)

\[
i \cdot \frac{1}{s^0} + \frac{1}{v_s} \cdot 0.75 \cdot u^q(c_s) = u^q(1_i \cdot c_s) \frac{1}{s^0}
\]

and using the ...rst order condition with respect to \( v_s \), one arrives at:

\[
i \cdot V^0(v_s) u^q(c_s) = u^q(1_i \cdot c_s)
\]

Differentiating we obtain

\[
i \cdot V^q(v_s) u^q(c_s) dv_s \cdot i \cdot V^0(v_s) u^q(c_s) dc_s = i \cdot u^q(1_i \cdot c_s) dc_s
\]

which implies that \( dc_s = dv_s > 0 \) as \( V^0 < 0, V^q < 0, u^q < 0 \). Therefore also consumption to agent 1 rises. Furthermore, from the binding (IR1) constraint \( u(c_s) + \bar{v}_s = u(y_s) + -V^{aut} \) and the fact that \( v_s > V^{aut} \), it follows that \( c_s < y_s \), i.e. although consumption for agent 1 will increase, agent 1 will give up some current consumption in exchange of future consumption to insure agent 2. In conclusion, when agent 1 receives a very good shock, the planner gives him more consumption, but by promising him also higher future consumption, she induces agent 1 to insure partially agent 2 who, in turn, has a fall in consumption, but not as big as in autarky. In this situation, the pair \( (c_s, v_s) \) solves

\[
i \cdot \frac{u^q(1_i \cdot c_s)}{u^q(c_s)} = V^q(v_s);
\]

\[
u(c_s) + \bar{v}_s = u(y_s) + -V^{aut}:
\]

Let us now try to characterize the level of income shock \( y \) such that, for \( y > \bar{y} \), we are in region 2. Rewrite the (IR1) constraint as

\[
u(c_s(v_s)) + \bar{v}_s = u(y_s) + -V^{aut}:
\]

The value \( \bar{y} \) is the cut-off between region 1 where \( v_s = \bar{v} \) and region 2, therefore \( \bar{y} \) solves

\[
u(c_s(v)) + \bar{v} = u(y) + -V^{aut}:
\]

Once we totally differentiate the expression above, we obtain \( \bar{y}^q(v) > 0 \), thus given that the planner enters the current period with a promise of utility \( \bar{v} \) to agent 1, the larger is \( \bar{v} \) the higher has to be \( y_s \) in order to make the value of autarky attractive compared to the contract.
3. Finally, we have a third case, when the endowment $y_s$ is very low. The participation constraint (IR2) binds and $s = 0; \frac{1}{s} > 0$. It is straightforward to show that this case is exactly symmetric to the previous one, i.e. the planner increases both current consumption and future utility to agent 2 and gives some insurance to agent 1. The pair $(c_s; v_s)$ solves

$$\frac{u'(1; c_s)}{u'(c_s)} = V'(v_s);$$

$$u(1; c_s) + V(v_s) = u(1; y_s) + V^{aut};$$

We can summarize the solution in the following graph in the $(c_s; y)$ space. We have three regions: the intermediate region where $y_s \approx y^*$ is the perfect insurance region, with consumption independent of income shocks. None of the IR constraints are binding and the planner can offer perfect insurance (there are small incentive problems, as both agents get similar shocks). In the other two regions, where $y_s < y^* \approx y$, the incentive problems dominate the desire of the planner to provide full insurance. One of the two (IR) constraints dominate, insurance is only partial and individual consumption rises with income.

1.4 An Example: From Income to Consumption Inequality (Krueger-Perri, 2001a)

Consider a simple version of the Kocherlakota model, where the aggregate endowment equals 2 and the endowment shock follows the symmetric process

$$y_i = \begin{cases} 1 + \frac{3}{4} & \text{with Pr} = 1/2 \\ 1 - \frac{3}{4} & \text{with Pr} = 1/2 \end{cases}$$

which has mean 1 and standard deviation $\frac{3}{4}$. We consider values of $\frac{3}{4} (0; 1)$, since the aggregate endowment is exactly 2: One can easily compute the value of autarky to be

$$V^{aut} = \frac{u(1 + \frac{3}{4}) + u(1 - \frac{3}{4})}{2(1 - \bar{\nu})};$$

and the values of defaulting after a low and a high realizations of the shock to be $U(1 - \frac{3}{4}) = u(1 - \frac{3}{4}) + \bar{V}^{aut}$ and $U(1 + \frac{3}{4}) = u(1 + \frac{3}{4}) + \bar{V}^{aut}$. The full risk-sharing arrangement (the unconstrained best) is equivalent to getting $u(1)$ forever, with associated utility

$$U^* = \frac{u(1)}{1_i};$$

It is useful to characterize the properties of the function $U$. Notice that when $\frac{3}{4} = 0$, $U(1) = U^*$, i.e. without income fluctuations, being in autarky is equivalent to full risk sharing. Moreover, given the strict concavity of $u$, $U$ is also strictly concave. How do the $U$ functions depend on $\frac{3}{4}$? Start from $U(1; \frac{3}{4})$:

$$\frac{\partial U(1; \frac{3}{4})}{\partial \frac{3}{4}} = u'(1; \frac{3}{4}) + \frac{u(1 + \frac{3}{4})}{2(1 - \bar{\nu})} - \frac{u'(1; \frac{3}{4})}{2(1 - \bar{\nu})} < 0;$$

where the strict inequality follows from the strict concavity of $u$. Therefore, for any realization of $\frac{3}{4}$ full risk sharing is always preferred to defaulting when the agent has a low income realization, i.e. $U^* > U(1; \frac{3}{4})$ for any $\frac{3}{4}$ Consider now $U(1 + \frac{3}{4})$:

$$\frac{\partial U(1 + \frac{3}{4})}{\partial \frac{3}{4}} = u'(1 + \frac{3}{4}) + \frac{u(1 + \frac{3}{4})}{2(1 - \bar{\nu})} - \frac{u(1 + \frac{3}{4})}{2(1 - \bar{\nu})} < 0;$$

$$\frac{\partial U(1 + \frac{3}{4})}{\partial \nu} = u'(1) > 0.$$
where the inequality for $\frac{\gamma}{\bar{r}} = 1$ comes from the assumption that u satisfies the Inada conditions, so $u(0) = -1$. Since U is strictly concave, $U(1 + \frac{\gamma}{\bar{r}})$ it will have a unique maximum at $\frac{\gamma}{\bar{r}}$, which lies above $U^a$ and it will cross $U^b$ once at $\frac{\gamma}{\bar{r}}$. The non-monotonicity of $U(1 + \frac{\gamma}{\bar{r}})$ depends on two contrasting forces: the higher is $\frac{\gamma}{\bar{r}}$, the larger is the income-effect that makes defaulting attractive, but at the same time a high $\frac{\gamma}{\bar{r}}$ makes autarky very risky (risk-effect). For low (high) values of $\frac{\gamma}{\bar{r}}$, the (income) risk effect prevails. Figure 2 shows the U functions in the $(U; \frac{\gamma}{\bar{r}})$ space.

Kehoe and Levine (2002) prove that the solution to the constrained efficient problem is completely characterized by a number $c(\gamma/\bar{r})$ such that agent 1 consumes $1 + c(\gamma/\bar{r})$ and agent 2 consumes $1 + c(\gamma/\bar{r})$. In other words, promised utility $v_s$ in this stationary example with 2 shocks is not a state variable: given a consumption allocation $c$, we can write promised future utility independently of the current realization of the shock as $v = u(1 + c) + u(1 + \frac{\gamma}{\bar{r}})$; hence $U(1 + c) = u(1 + c) + \frac{\gamma}{\bar{r}} v$.

It can also be proved (and it is once again very intuitive) that $c(\gamma/\bar{r})$ is the smallest non-negative solution to the equation:

$$U(1 + c) = \max_f U^a; U(1 + \frac{\gamma}{\bar{r}})$$

To understand, consider two cases:

1. If $\max_f U^a; U(1 + \frac{\gamma}{\bar{r}})g = U^a$; then the economy is at a level of the standard deviation of income shocks so high that even with the best realization, full risk sharing is always preferred to defaulting, hence $c = 0$ and both agents consume 1 forever. This region will arise for $\frac{\gamma}{\bar{r}} < \frac{\gamma}{\bar{r}}$.

2. If $\max_f U^a; U(1 + \frac{\gamma}{\bar{r}}) g = U(1 + \frac{\gamma}{\bar{r}})$ then defaulting is more attractive than full insurance. Hence, the planner cannot implement the first best and at most we can have a solution with partial consumption insurance, i.e. $0 < c < \frac{\gamma}{\bar{r}}$. In some cases autarky will prevail with $c = \frac{\gamma}{\bar{r}}$. We have always two solutions to the equation $U(1 + c) = U(1 + \frac{\gamma}{\bar{r}})$. One is always the autarkic allocation $c = y$. Graphically, it is easy to see that when $\frac{\gamma}{\bar{r}} > \frac{\gamma}{\bar{r}}$ we also have another smaller non-negative solution $c(\gamma/\bar{r})$ that provides exactly the same utility $U(1 + \frac{\gamma}{\bar{r}})$ when the individual receives a high realization, but larger utility when the realization is low, so this partial insurance agreement is preferred to autarky. However, for $\frac{\gamma}{\bar{r}} <= \frac{\gamma}{\bar{r}}$ autarky is preferred and no risk sharing is possible: intuitively, risk sharing implies a transfer from the rich to the poor households and for $\frac{\gamma}{\bar{r}} <= \frac{\gamma}{\bar{r}}$ if the planner takes income away from the rich agent, the latter's utility in the high state never increases (the income effect is dominant).

The three regions (autarky, partial insurance, full risk sharing) are described in Figure 2. At this point is simple to answer the following question: what happens to consumption inequality when there is a rise in income inequality? The answer depends on the initial point. If the economy is in the autarky case, then consumption inequality rises one for one (unless there is a switch of regions). If the economy is in the full-insurance region, consumption inequality is always zero and does not respond to income inequality. The interesting case is the middle region with partial risk sharing. A rise in $\frac{\gamma}{\bar{r}}$ reduces consumption inequality. The mechanism is simple: since the value of autarky falls with $\frac{\gamma}{\bar{r}}$ (because of strict concavity of preferences) the individual is less attracted by the autarkic allocations and engages in more risk sharing, so $c$ falls and so does consumption inequality.

1.5 The Role of Public Insurance (Krueger-Perri, 2001b)

Let's now introduce a government that redistributes from rich to poor (i.e. from the high income states to the low-income states) through taxation and lump-sum transfers. We maintain that the only enforcement technology for private risk-sharing contracts is exclusion from all future participation to
...nancial markets, while tax liabilities are not subject to this enforcement problem: even if an agent ends up in autarky, the government will collect her taxes.

Registutative taxation in this class of economies have two ects: 1) a direct eect of reducing income fluctuations and insuring agents by redistributing from good to bad states, and 2) an indirect eect of increasing the value of autarky (as the worst possible state is made now more acceptable), with the consequence of reducing the amount of private risk-sharing in the economy. So there is a trade-off between the increase in public insurance and the fall of private insurance which is crowded-out by government redistribution.

Note that here all the classical incentive problems linked to distortions of labor supply and capital accumulation are left out, and in fact in a standard endowment economy with exogenous incomplete market (a la Huggett), a positive amount of redistribution is unambiguously good. The analysis of the eects of taxation in limited enforcement economies is more sophisticated that in an economy with exogenous incomplete markets because, as suggested originally by Stiglitz, when one analyzes the impact of taxation as a solution for some form of market imperfection, it is crucial to specify what is the information/transaction problem at the source of incompleteness, as this same source of incompleteness could be worsened by the policy.

Krueger and Perri consider the switch from a proportional to a progressive tax system, for a given amount of government transfers. They highlight two cases. First, take an economy with very little private risk-sharing, where most of the insurance is provided by taxes. There the shift to progressive taxation has very little indirect eects, but large positive direct eects through redistribution: ex-ante welfare rises substantially. One can easily see that this case roughly corresponds to the autarky region of Figure 2. The change in policy corresponds to a fall in $\sigma$ (standard deviation of post-tax endowment), which unambiguously increases ex-ante welfare, i.e. $[U(1 + \frac{1}{4} + U(1 - \frac{1}{4})] \geq 2$ rises. Second, consider an economy with some amount of private risk-sharing, i.e. the region ($\frac{1}{2}, \frac{3}{4}$) of Figure 2. The shift towards more redistribution would reduce $\sigma$ and as we have explained already, decrease the amount of private risk sharing, decreasing ex-ante welfare: the crowding out eect on private insurance markets is larger than the direct eect.

Homework: what is the optimal tax policy, i.e. the level of taxes and transfers that can implement the rst best? Is there only one or more than one?

1.6 Implications of Limited Enforcement Models

The limited enforcement model has a number of interesting implications (some of which are testable with available data):

Individual consumption is not a random walk, it depends on current income shocks, in particular for very high and very low realizations of income shocks. Therefore the model can account for the so-called “excess sensitivity puzzle”. One way to test this implication is to specify a nonlinear (of the threshold type) time-series model, where $c_i = f_t(y_i)$, i.e. the function $f$ is nonstationary and depends on the time $t$ realization of the income shock $y_i$: it will be constant for intermediate levels of the shock and monotonically increasing for low an high levels.

The lack of commitment induces persistence in individual consumption even if the shocks are iid. Consider agent 1 when he receives a high realization of $y_i$, so the (IR1) is binding. Then we have that both $c_1$ and $v_2$ increase over time, i.e. it is not optimal from the point of view of the planner to give the extra utility only through current consumption, but some of it is also given through higher future consumption. Thus, the eects of a high iid shock persist over time.

Consider condition (7) that equates marginal rates of substitutions between agents. That condition states that (a function of) the ratio $c_2 = c_1$ is a su cient statistics for $v_2$ and therefore for the
future evolution of individual consumption. With \( N \) agents, the vector of all ratios of individual consumption is the sufficient statistics. One way to test this implication is running a regression of individual consumption \( c_{t+1} \) on the \( N \) dimensional vector of lagged consumption ratios and other lagged variables.