1 Optimal Unemployment Insurance

Why should a government be concerned about providing insurance to unemployed workers? The presumption is that unemployment leads to a sharp fall in consumption and welfare that cannot be insured away otherwise. Browning and Crossley (2001) and Gruber (1997) measure the average loss of consumption for workers unemployed at least 6 months to be around 14%. Grueber measures that for the average worker a rise of 10% in the replacement ratio of unemployment insurance (UI) would reduce the fall in consumption by 2.5%: this means that eliminating completely UI, i.e. going from an average replacement ratio of 50% to zero, would increase the consumption loss substantially, by over 12%. However, Browning and Crossley make clear that behind this average number it lies a lot of heterogeneity. For workers with positive assets, the marginal effects of UI are zero, i.e. these workers can self-insure almost perfectly; only workers without assets benefit from UI. Interestingly, Gruber finds that UI crowds-out other forms of insurance (capital accumulation, intrafamily transfers and labor supply of the second earner). In particular, a 10% rise in the replacement ratio of UI reduces the saving rate by 7%, while the effects on other sources of insurance are much smaller and often not strongly significant.

Our aim in this section is to study the problem of a benevolent government whose objective is to reduce the costs from being unemployed associated to the sudden loss of labor income and consumption for an household who has no other source of self-insurance, under the constraint that public expenditures on this welfare program should be minimized. The government needs to give insurance to the individuals (through transfers of income) in order to allow them to keep their consumption from falling too much, but it also must give them incentives to provide search effort and exit unemployment quickly (to minimize the expenditures). The key trade-off is therefore between insurance and incentives.

In the benchmark model, an agent has preferences

\[ E_{t=0} \sum_{i} [-u(c_i) + a_i]; \]

where \( c_i \) is consumption, with \( u^0 > 0, u^{00} < 0 \) and \( u(0) \) finite. The choice variable \( a_i \in [0,1] \) is search effort, whose disutility is assumed to be linear. Note also that preferences are separable in consumption and effort. The uncertainty implicit in the expectation operator \( E_{0} \) is on the future employment status. Conditional on being employed, the worker is paid \( w \) and employment is an absorbing state, i.e. \( V^e = w(1 - \bar{p}) \). An unemployed worker who searches with effort \( a \) has probability of finding a job \( p(a) \); with \( p^1 > 0 \) and \( p^{00} < 0 \). There is no storage technology, i.e. no means of self-insuring for the individual, only a government (insurance agency) that offers an insurance contract.

1.1 Autarky

Consider first the case of autarky, where the government is absent and no insurance contract is offered. The problem of the unemployed individual in dynamic programming form is

\[ U = \max_a \left[ u(0) + \sum_{i} [-p(a)V^v_i (1_i - p(a)) U_i] \right] \]

with associated first order condition

\[ 1 = -p^0(a_{\text{aut}}) [V^v_i U] \]

which tells us that in autarky the household chooses optimally her search effort so that its marginal disutility equals exactly the expected marginal gain from searching slightly more intensively. The solution of the problem (1) consists of a pair \( (a_{\text{aut}}; U) \) that can be obtained easily by starting from a
generic $U^0$ and iterating over the two equations

$$1 = -p(a^n) \frac{W}{\bar{a}_1} U^n,$$

$$U^{n+1} = u(0) + a^n + \frac{W}{\bar{a}_1} (1_i, p(a^n)) U^n. \quad (3)$$

### 1.2 Unemployment Insurance with Full Enforcement

Suppose now that the government can perfectly observe the search effort of the agent and therefore can fully enforce it: the government can elicit any amount from the worker, exactly in the same way it commands tax payments. The government (like a planner) minimizes the costs of the welfare program and chooses a level of expenditures $c$ (consumption of the unemployed agents), a level of search effort for the individual $a$, and promises future utility $v$, all subject to a promise-keeping constraint, i.e.

$$W(\psi) = \min_{c,a,v} f \psi + (1_i, p(a)) W(\psi) g$$

s.t.

$$u(c) + a - [p(a)V^e + (1_i, p(a)) v], \quad \psi \quad (PK)$$

where $W(\psi)$ is the cost function for the planner who promised utility $\psi$; and $W(\psi) > 0$. For the minimization problem to have a unique solution, $W(\psi)$ needs to be strictly convex.

The FOC with respect to promised utility $\psi$ gives $W(\psi) = 3$; where $3$ is the multiplier on the $(PK)$ constraint. The envelope condition yields $W(\psi) = 1$, from which it follows that $v = \psi$, thus promised utility is constant along the unemployment spell, and so is the multiplier $3$. Since the FOC with respect to consumption is simply $1 = \frac{1}{3} u_0(c)$; it follows immediately that also consumption is constant along the unemployment spell, hence the government will give full insurance to the individual unemployed worker and the first best is reached.

What about search effort? With full enforcement, the FOC with respect to $a$ is

$$-p(a)W(\psi) = (1_i, p(a)) (V^e, v);$$

which, once we use the fact that $v = \psi$ and the envelope condition $W(\psi) = 1$, becomes

$$(V^e, v) + \frac{W(\psi)}{W(\psi)} = \frac{1}{p(a^n)}; \quad (3)$$

It is straightforward to compare $(3)$ to the first order condition for search effort under autarky $(2)$. Notice that $W(\psi) = W^0(\psi) > 0$, and $p^0 < 0$. Therefore, this comparison tells us immediately that $a^F > a^{aut}$. Intuitively, the worker who has $c$ guaranteed as consumption when unemployed would have an incentive to exert less effort if she could choose optimally without the intervention of the government, as she does not internalize the costs of the scheme. The higher effort is the price the worker has to pay to be able to smooth consumption more effectively than in autarky.

### 1.3 Unemployment Insurance with Limited Enforcement

We now make a more reasonable assumption on what the government can and cannot observe. Suppose that, as in actual economies, government agencies can provide insurance but they cannot perfectly

\[1\] Recall that since the problem is a minimization problem, the Lagrangean is written as the objective function minus the multiplier $\lambda$ times the $(PK)$ constraint, with $\lambda > 0$. 

observe the effort of the unemployed worker. The planner’s problem now reads

\[
W(v) = \min_{c,v} \left[ c + \frac{1}{1 - p(a)} \left[ p(a) V_e + (1 + p(a)) v \right] - p(a) (V_e - v) \right]
\]

s.t:

\[
u(c) - a + \frac{1}{1 - p(a)} \left[ p(a) V_e + (1 + p(a)) v \right] = \frac{1}{1 - p(a)} \]

The last constraint states that the effort is chosen independently by the unemployed agent, hence she will do so optimally as if she were in autarky, i.e. following the rule (2). Denote by \( \bar{\lambda} \) the multiplier on the (IR) constraint.²

The FOC with respect to \( c \) is always \( 1 = 1 + u'(c) \), but now the choice of future utility \( v \) is characterized by the condition

\[
- (1 - p(a)) W'(v) + 1 \left[ (1 + p(a)) +, p(a) \right] = 0
\]

Thus:

\[
W(q(v)) = 1 + \frac{p(a)}{1 + p(a)} W'(v) + \frac{p(a)}{(1 + p(a))},
\]

where in the last equation we have used the envelope condition \( 1 = W(q(v)) \). Given the convexity of \( W(v) \); we have that \( v < v \). Therefore \( 1 \) decreases over the unemployment spell and so does \( c \) since \( 1 = 1 + u'(c) \). In other words, the government promises less and less future utility to the unemployed worker (i.e. decreases the transfer of income over the unemployment spell) to increase the difference between the value of employment and the value of unemployment \( V_e - v \) and induce her to increase her search effort, through the optimality condition (2).

In conclusion, the optimal unemployment insurance scheme with the assumption of limited commitment is designed as a government transfer that decreases over time, similarly to many actual schemes that we observe in a number of developed countries.

²Recall that since the problem is a minimization problem, the Lagrangean is written as the objective function minus the multiplier \( 1 \) times the (PK) constraint, minus the multiplier \( , \) times the (IR) constraint, with \( 1 > 0 \) and \( > 0 \).