MA2 - Advanced Macroeconomics: Homework 1
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The assignment is due on Wednesday, February 14 in class.

Exercise n. 1
Consider the Cass-Koopmans optimal growth model with a government who designs the fiscal policy in the economy. The household enjoys utility from two goods, private consumption $c$ and public consumption $g$. Preferences are separable over the two goods and, in particular, they are CIES over private consumption, with risk-aversion parameter $\sigma$ and discount rate $\rho$:

$$U(c, g) = \int_0^\infty e^{-\rho t} \left[ c(t)^{1-\sigma} \left( \frac{1}{1-\sigma} + v(g(t)) \right) \right] dt$$

The aggregate technology is Cobb-Douglas with share of capital equal to $\alpha$ and CRS. Households supply inelastically one unit of labor at the current wage rate $w(t)$ and accumulate assets which yield pre-tax return $r(t)$. The government sets taxes on capital income $\tau$ and can also issue bonds $b(t)$ with perpetual maturity $r_b(t)$ to finance the exogenously given pattern of government expenditures $\{g(t)\}_{t=0}^\infty$.

1) Define rigorously a competitive equilibrium for this economy. Is this an economy where the welfare theorems hold? Why?
2) Suppose first that $g(t)$ is financed only through taxes on capital income, because the government has to balance the budget every period.
   2a) Does the $\exists c=0$ locus change compared to the standard economy with no government (i.e. no expenditures and no taxes)?
   2b) What is the effect of an unexpected permanent increase in government expenditures on $k$ and $c$? Use the phase diagram for your answer and draw the time-path of consumption from the time of the announcement of the tax raise, until consumption reaches the new steady-state level.
3) Suppose now that $g(t)$ is financed through taxes on capital income, but the government can also run budget deficits (not forever, though!!!). Does Ricardian equivalence hold in this case? Prove formally your answer.

Exercise n. 2
Consider an economy with 2 consumption goods, durables $d$ and services $c$, and no capital good. Preferences over $(c, d)$ and hours of labor supplied are given by:

$$u(c, d) = \left[ \frac{c + d}{1-\sigma} \right]^{1-\sigma} - c \alpha l$$

The consumer offers labor services at market clearing rate $w$. The government finances expenditures $g$ through taxes on the two consumption goods at rates $\tau_c$ and $\tau_d$ and balances its budget every period. Firms produce the two goods with CRS technology $f(l) = l$ and sell them at market clearing prices $p_c$ and $p_d$.

1) Define a competitive equilibrium, and a Ramsey equilibrium for this economy.
2) Solve for the static Ramsey problem of the government and show that both consumption taxes have to be set to the same constant value.
3) What is the analogy with the dynamic Ramsey problem where the government has to set optimally labor taxes over time?

Exercise n. 3
Consider the same economy we described in the lecture, with the only difference that the government has a different set of tax instruments available: instead of labor income and capital income taxes, the government can tax labor income at rate $\tau(t)$ and consumption at rate $\phi(t)$.

1) Define a competitive equilibrium and derive all the conditions of the Maximum Principle for the consumer’s problem.
2) Define a Ramsey equilibrium and solve the Ramsey problem with the right feasibility and implementation constraints.
3) How should labor and consumption taxes be set from period $t > 0$ on?
4) In the light of your answer to question 3), how would you judge a proposal of a balanced budget amendment which forbids by constitution the government to run temporary deficits?