Exercise n. 1
Consider the Euler Equation for household consumption that we have derived in class
\[ c_t = \frac{1}{1 + \rho} E_t \left[ (1 + r_{t+1}) c_{t+1} \right], \]
which can be written as
\[ 1 + \rho = E_t \left[ (1 + r) (1 + g_c)^{-\varphi} \right]. \]

1) Take a second order Taylor approximation around \( r = g = 0 \), drop all the square terms (assuming they are small), assume that the rate of return is on a risk-free asset, so \( E_t [r] = \frac{\theta}{2} \), and derive the equation
\[ E_t [g_c] = \frac{1}{\rho} \left( \frac{\theta}{2} \right) + \frac{1}{2} \text{var}(g_c). \] (1)

2) Argue that without uncertainty, we go back to the standard consumption growth equation.

3) From the WWW, look for data on aggregate consumption and real interest rate for a country of your choice for which a long time series is available (ideally, quarterly data for 40-50 years) and estimate the empirical equivalent of equation (1). Comment on the results of your estimation, with particular emphasis on the importance of aggregate precautionary savings.

4) Can you detect anything different in the relationship between the first part and the second part of the sample (e.g. split the estimation in two periods, say before and after 1980).

Exercise n. 2
Consider the following Euler equation for consumption, where it is assumed that \( r \) is the return on the risk-free asset:
\[ c_t = \frac{1}{1 + \rho} (1 + r) E_t \left[ c_{t+1} \right]. \]

1) Assuming that the distribution of \( c_{t+1} \) is lognormal, derive the relation between expected consumption next period and consumption this period, and show how it depends on the interest rate, the discount rate and the variance of next period consumption. Explain your answer.

Exercise n. 3
Let \( H \) denote the stock of housing in London, with \( \delta \) its depreciation rate, \( I \) the investment in housing, \( p \) the price of a house and \( R \) the rent. Assume that \( I \) is increasing in \( p \) (the higher is the price, the more builders construct houses), and that the rent \( R \) is a decreasing function of the stock of housing available. Finally assume that there is another investment opportunity (i.e. shares) that gives rate of return \( r \), so a no arbitrage condition requires
\[ \frac{R + \delta}{\rho} = r. \]

1) Determine the locus \( H = 0 \) and the locus \( p = 0 \) in the \((H, p)\) space. Study the dynamics of the system and explain your intuition. Is it a saddle path? Why is the \( H = 0 \) locus not horizontal, like it was in the \( q \) model?

2) Suppose the London housing market is in steady-state and there is a stock-market boom, so \( r \) rises. What happens to the new long-run equilibrium? How do \( H, p, I, R \) adjust along the dynamics?

3) Suppose that, following a season of unusually persistent rain, the river Thames overflows and destroys a fraction \( \alpha \) of the housing stock. What are the equilibrium dynamics of the housing market following this shock?