MC2 - Core Macroeconomics: Homework 5
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The assignment is due Monday November 6th. To be handed in class at 3:00 pm to Nick Rau.

Exercise n. 1

Consider Romer’s endogenous growth model, and solve the associated Social Planner’s problem for \( g^* \), the optimal growth rate of the economy.

1) Compare \( g^* \) with \( g \), the growth rate of the economy under the decentralized equilibrium, and interpret the difference. Where do the two distortions of Romer’s economy (monopolistic competition and externality in \( R&D \)) show up?

Suppose a government want to correct these distortions by means of subsidies to firms financed by lump sum taxation to households. Obtain the equilibrium growth rates \( g \) with a subsidy \( \theta \) to:

2) production of final good \( y \)
3) purchase of each intermediate good \( x(i) \)
4) labour costs \( w_A \) in the research sector
5) Could any of the above subsidies modify agents’ incentives in order to achieve exactly the optimal growth rate \( g^* \)?

Exercise n. 2

Consider Romer’s model with the following twist. The firm-level production function of \( R&D \) is:

\[
\dot{a} = \delta (A^\beta L_A^{\lambda-1}) L_A.
\]

with \( 0 < \beta, \lambda \leq 1 \). The meaning of this law of motion is that: first, past knowledge \( A \) does not increase future knowledge at a linear rate, but it has decreasing returns; second, at the level of the \( R&D \) sector also labor input \( L_A \) has decreasing marginal returns, because different firms might duplicate research activities and come up with the same idea. Thus, at the level of the sector, after aggregating:

\[
\dot{A} = \delta (A^\beta L_A^{\lambda-1}) L_A = \delta A^{\beta} L_A^\lambda.
\]

Assume also that \( L \) grows at a constant rate \( n \), exogenously.

1) Solve the model with these new assumptions and obtain the equilibrium growth rate \( g \).
2) Discuss the main differences between this growth rate and the one in the standard Romer’s model.
3) Do we still have an endogenous growth model?