Consumption and Labor Supply with Partial Insurance: An Analytical Framework

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Preliminary and incomplete draft: October 2006

Abstract
This paper develops an analytical framework to study consumption and labor supply in a rich class of heterogeneous-agent economies with incomplete markets. The environment allows for trade in non-contingent and state-contingent bonds, for permanent and transitory idiosyncratic productivity shocks, and for permanent preference heterogeneity and idiosyncratic preference shocks. Exact closed-form solutions are obtained for equilibrium allocations and for the first and second moments of the equilibrium joint distribution over wages, hours and consumption. With these expressions in hand, we show that all the structural preference and risk parameters in the model can be identified, even when productivity risk varies over time, given panel data on wages and hours, and cross-sectional data on consumption. We structurally estimate the model on CEX and PSID data for the U.S. economy for the period 1967-1996. We then use the estimated parameter values to decompose inequality in all variables of interest, both over the life-cycle and across time, into cross-sectional variation in preferences, uninsurable wage risk, insurable wage risk, and measurement error.

The authors thank Greg Kaplan for outstanding research assistance and the Federal Reserve Bank of Minneapolis for its hospitality. Heathcote and Violante’s research is supported by a grant of the National Science Foundation (SES 0418029). The views expressed here are those of the authors and do not necessarily reflect those of the Federal Reserve System.
1 Introduction

The transformation that occurred in the U.S. wage structure over the last thirty years gives economists a rare opportunity to deepen our understanding of how individual productivity shocks transmit to consumption, leisure and, eventually, to household welfare. This transmission mechanism is mediated by three key factors: preferences, availability of insurance, and advanced information about productivity changes. Willingness to substitute intertemporally and labor supply flexibility amplify the impact of income shocks on consumption and leisure. Insurance channels, such as credit markets, within-group risk-sharing, family labor supply, and government transfers moderate the effect of individual shocks on household consumption. A priori knowledge of the evolution of earnings over the life-cycle allows individuals to smooth consumption more effectively, vis-a-vis changes in lifetime earnings. A large literature, discussed below, has taken different approaches to organizing the data in order to learn about all these issues.

This paper develops a new equilibrium framework to study consumption and labor supply decisions in an economy where heterogeneous households have only partial insurance against idiosyncratic labor productivity shocks. The key advantage of our model, compared to those explored in the existing literature, is that it is analytically tractable (i.e., it requires no numerical solutions and no analytical approximations), and yet is rich enough to include a number of realistic features such as flexible preference specifications, permanent and transitory productivity shocks, permanent and transitory heterogeneity in the taste for leisure, and within-group insurance over and above that provided by trade in a single risk-free asset. The model yields exact closed-form solutions for all the first and second cross-sectional moments of the joint equilibrium distribution of wages, hours worked, and consumption as a function of all the structural parameters. Thus, the mapping between the evolution of inequality and basic preferences and technologies is transparent. We exploit this transparency by first compiling a large set of empirical moments characterizing the profiles for cross-sectional dispersion over the life-cycle and across time in the United States, and then using the corresponding model expressions to extract useful information from these facts about the persistence and insurability of wage risk, the role of preference elasticities in the transmission of shocks, and on relative importance of preference and productivity shocks in accounting for cross-sectional variation.
in consumption and leisure.

The strategy to maintain tractability is a substantial generalization of the results in Deaton (1991), and Constantinides and Duffie (1996). Deaton (1991) showed that, in an economy where a risk-free bond is the only financial asset, agents cannot borrow to smooth out permanent income shocks without violating the budget constraint. Thus, for a sufficiently low interest rate, they would slowly run down their assets to zero, and then consume their earnings every period. Constantinides and Duffie (1996) proved that this latter scenario is an equilibrium (“no bond-trading” equilibrium): at the right interest rate, the expected change in the marginal utility of consumption is equalized across all agents in the absence of bond trade. Thus the bond is not traded, even though it is available. As a result, agents’ optimal wealth-holdings are always zero. This means that wealth can be dropped as a state variable in the household’s problem, and hence the model becomes analytically tractable. We show that this result holds under three important generalizations that allow the model to be better equipped to confront the data: first, the introduction of flexible labor supply; second, the introduction of a second, orthogonal productivity shock that is fully “insurable”; third, the introduction of (insurable) preference shocks and (uninsurable) preference heterogeneity in the taste for leisure relative to consumption.

The first contribution of the paper is “qualitative”: our framework allows a degree of transparency in the analysis of equilibrium allocations of heterogeneous agents economies with incomplete markets that has no precedent in the literature. We give two examples here. First, we can derive in closed form a simple expression for the welfare effect of a change in labor market risk, as a function of elasticities, variances of insurable and uninsurable shocks, and preference heterogeneity. Second, we show how the theoretical closed-form expressions for second moments by year and cohort can be used to identify all the deep preference and (uninsurable and insurable) risk parameters, even when the latter are allowed to change over time in an unrestricted manner.

The other contribution is “quantitative”. The estimation of the model, performed with a Minimum Distance Estimator, allows one to answer quantitatively some important questions concerning the effect of individual wage shocks on household consumption, leisure and welfare. What is the size of key elasticity parameters, such as the intertemporal elasticity of substitution and the Frisch labor supply elasticity? What fraction of the
recent rise in wage dispersion was insurable from the standpoint of U.S. households? What is the relative importance of productivity versus preference variation in accounting for heterogeneity in consumption and labor supply outcomes across households?

The existing literature has taken several alternative approaches, all fruitful in their own way, to investigate how the recent changes in the wage structure transmitted to household consumption and welfare. The first approach, which one may call “empirical” (Attanasio and Davis, 1996, Krueger and Perri, 2005) uses a minimum amount of guidance from the theory to test (and reject) the complete markets hypothesis and compute the welfare costs of the rise in labor market risk. The “semi-structural” approach (Blundell and Preston 1998, Attanasio, Berloffa, Blundell, and Preston 2002, Blundell, Pistaferri and Preston 2004) builds on the individual consumption-saving problem under the PIH and uses jointly consumption and income data to identify the fraction of shocks which is uninsurable. With respect to these two approaches we share the view (originally advocated by Deaton, 1977) that, given the multiplicity and complexity of insurance channels potentially available to households, a useful first step is to quantify the overall degree of insurability, remaining somewhat agnostic about the sources. The value of taking this route is twofold. First, it establishes a benchmark (in terms of amount of insurance) that investigations with more explicit models of risk-sharing should attain. Second, it delivers an estimate of a parameter (the size of individual “consumption innovations”) that is key in the design of optimal taxation in the presence of incentive problems (Farhi and Werning, 2006).

The research program pursued by Cunha, Heckman and Navarro (2005) and Cunha and Heckman (2006) is close in spirit to these studies. These papers try to separate uncertainty (i.e., unforecastable shocks) from heterogeneity (i.e., information known by agents at the time of their decisions) as a determinant of earnings and consumption dispersion (see also Guvenen, 2005). A major theoretical challenge, emphasized by the authors, is to nonparametrically simultaneously identify preferences, market structures, and information structures. By making fairly general parametric assumptions about preferences and shocks, our work allows one to separately identify preference heterogeneity, preference shocks, and the scope of insurance markets. At the moment our work has little to say about households’ “information sets”. We discuss this issue further in the conclusions.

The “fully structural” approach (Krueger and Perri, 2006, Heathcote, Storesletten and Violante, 2004) lays out a very explicit economic environment, by making specific
assumptions on the statistical properties of the wage shocks, market structure, and informal channels of insurance (e.g., flexibility of labor supply). These exercises can be interpreted as “joint tests” of all the assumptions, as well as ways to quantify the extent to which a particular artificial economy (e.g., a life-cycle economy with labor supply and self-insurance) can account for the changes in the various dimension of cross-sectional dispersion in the U.S., over the last three decades.

The approach in this paper shares the emphasis of previous structural work on laying out a general equilibrium model of partial insurance: in our framework, the interest rate adjusts as the variance of insurable and uninsurable shock changes over time. However, one important difference in that equilibria in most fully structural model economies can only be computed numerically, because the wealth distribution is part of the fixed point problem. Even though extensive comparative statics can be performed to try to disentangle forces at work, this limits the limpidity of the analysis. Our closed-form expressions can shed additional light on the interactions between preferences, persistence of shocks and market structure that are implicit in numerical simulations.

The rest of the paper is organized as follows. Section 2 presents the cross-sectional facts that motivate this investigation. Section 3 develops our equilibrium framework, and derives analytically the allocations. Based on the allocations, in Section 4 we compute the closed-form expressions for all the equilibrium cross-sectional moments of interest. Section 5 proves how the various cross-sectional moments allow to identify the structural parameters of the model. Section 6 describes the estimation algorithm, and reports the results. Section 7 concludes the paper.

2 Facts

We begin by describing the sources of our micro-data and the selection criteria we imposed to construct our final sample. Next, we report a comprehensive set of facts about the evolution of cross-sectional dispersion of wages, hours, earnings and consumption both over the life-cycle, and over time. The model we develop in Section 3 is designed to account for all these facts jointly.
2.1 Data

Our data are drawn from two data sets, the *Michigan Panel Study of Income Dynamics* (PSID), and the *Consumer Expenditure Survey* (CEX). From the 1968-1997 waves of the PSID, we gather information on individual wages, hours and earnings. The PSID asks questions about earnings in the previous year, so our data refers to the period 1967-1996.\(^2\) We exclude observations from the Survey of Economic Opportunities (SEO), a sub-sample that over-represents poor households. The sample is hence representative of the US population, and sample weights are not used in any calculations.

The CEX data contain detailed information on nondurable and durable household consumption, a key input of our analysis. Consistent data over time are available only since the 1980 survey. As the CEX is not a representative sample, weights are used in all calculations. The starting point for our CEX sample is the same sample used by Krueger and Perri (2006). This includes all households who are complete income respondents and for whom we observe data from four consecutive quarterly interviews. We use two measures of consumption: one that excludes expenditures on durable consumption goods and one that comprises total household expenditure, including durables.

Since we will use all these data jointly, it is paramount to construct samples that are as comparable as possible. To this end, we impose exactly the same selection criteria across the two datasets. In every year, we select all heads of household (reference persons in the CEX) between ages 25 and 54, who are not full-time students, and whose annual market hours are between 520 and 5096, i.e., between one full-time week for a quarter and 14 hours a day, every day of the week. We drop observations if they are top-coded or if the hourly wage is below half the federal minimum wage in that year. In both datasets, the hourly wage is computed as annual pre-tax labor earnings divided by annual hours worked. All monetary variables are deflated using the Consumer Price Index (CPI) and expressed in 1992 dollars. The final PSID sample contains 73,678 individual-year observations, comprising 7,320 individuals of which 1,268 are present in the sample for at least 20 of the 30 possible years. The final CEX sample contains 34,060 household/year observations. Wages, hours and earnings always refer to the heads of household, while

\(^2\)The more recent waves of the PSID, after 1997, include questions about income only every second year and are hence excluded from the analysis.
Table 1: Sample selection

<table>
<thead>
<tr>
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<th>Sample Size</th>
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<tbody>
<tr>
<td></td>
<td>PSID</td>
</tr>
<tr>
<td>Baseline sample</td>
<td>290,375</td>
</tr>
<tr>
<td>Exclude SEO sample</td>
<td>154,297</td>
</tr>
<tr>
<td>Keep heads of households</td>
<td>89,400</td>
</tr>
<tr>
<td>Drop obs with missing earnings data</td>
<td>81,942</td>
</tr>
<tr>
<td>Drop top-coded earnings</td>
<td>81,858</td>
</tr>
<tr>
<td>Hours restrictions</td>
<td>79,021</td>
</tr>
<tr>
<td>Minimum wage restrictions</td>
<td>77,685</td>
</tr>
<tr>
<td>Age restriction</td>
<td>73,687</td>
</tr>
<tr>
<td>Drop full time students</td>
<td>73,678</td>
</tr>
<tr>
<td>Final Sample</td>
<td>73,678</td>
</tr>
</tbody>
</table>

both consumption expenditure measures are expressed in per adult-equivalent units in the household.\(^3\) Table 1 shows the number of observations lost at each stage of the selection process.\(^4\)

To filter out from the data the variation due to demographic factors that are outside the model, we regress individual log wages, hours and consumption (the former two in both datasets, the latter only in CEX) on year dummies, race (white, black or other) and gender dummies, and a quartic in age. We use the residuals from these regressions to construct variances and covariances of the joint distribution of wages, hours and consumption.

In what follows, we describe the evolution of the cross-sectional moments of interest both over the life-cycle and over time. When analyzed through the lens of our model, both life-cycle profiles and time-series dynamics of cross-sectional dispersion contain valuable information about preference parameters, as well as about the insurability of risk, and how the latter changes over time. We document facts about the joint distribution over the variables of interest (wages, hours, and consumption) both in levels and in first-differences. Macroeconomists working with heterogeneous-agents models have traditionally been more interested in the levels (e.g., Aiyagari 1994, Huggett 1996, Castaneda et al. 2003, Krusell

\(^3\)The equivalence scale is the same census equivalence scale used by Kreuger and Perri (2006).

\(^4\)A comparison across the two datasets in each year of the overlapping sample period 1980-1996 shows the proportion of college graduates, average earnings and hours worked are very similar in levels and they move closely over time. However, individuals are two years older, on average, in the CEX. The time series for the variance of hours in PSID tracks very closely the one in CEX. The same is true for wages, except for the last year in the sample, when CEX wages become much more dispersed than PSID wages. Overall, we conclude that the two samples are broadly consistent.
and Smith 1998). Labor economists, by contrast, have had a long-standing interest in studying the properties of these variables in first differences (e.g., Abowd and Card 1989, Blundell, Pistaferri and Preston 2004). For the sake of exposition, we will call the former the “macro facts”, and the latter the “micro facts”. In Section 5, we show that both sets of facts are needed to identify the structural parameters of our framework.

We now continue with the description of our findings, summarized in Figures 1-6.

2.2 Life-cycle facts

It is well known that to properly identify age-profiles, one has to take a stand on the presence of cohort and/or time effects in the data. In Heathcote, Storesletten, and Violante (2005a), we demonstrate that the data speak strongly in favor of time effects in the second moments, whereas we find little evidence of cohort effects. Consequently, in what follows, we control for time-effects. Effectively, we regress our second moments on a set of time-dummies and plot the residuals by age group, averaging across all cohorts. We group observations in six non-overlapping age classes: 25-29, 30-34, and so on until 50-54.

**Macro facts:** The variance of log wages increases by 14 log points between age 25 and 54, displaying a slightly concave pattern. The variance of log earnings increases as well, but by a smaller amount, roughly 11 log points. The variance of log hours is rather flat, thus what accounts for this smaller rise in the variance of log earnings is the fact that the covariance between hours and wages falls over the life-cycle.

The variance of log non-durable consumption grows mildly over the life cycle, by 4.5 log points. Looking at total consumption shows a larger rise, of about 6 points, with most of the increase occurring between ages 25 and 40. As we emphasized in Heathcote, Storesletten, and Violante (2005), these numbers represent much smaller increases than previously reported in the pioneering work of Deaton and Paxson (1994).\(^5\) The reason is that the Deaton and Paxson analysis, by controlling only for cohort-effects, implicitly incorporates rising dispersion over time into the age profiles. Moreover, their study covers the period 1980-1990, precisely when the bulk of the rise in cross-sectional dispersion is

\(^5\)Deaton and Paxson (1994, Figure 8) report an increase in the variance of log consumption of almost 20 log points between ages 25 and 55.
concentrated (see also Slesnick and Ulker 2004, for a related discussion), while our sample covers a longer time period.

Finally, the correlation between log consumption and log wages increases for most of the life-cycle, except for an initial drop, while the correlation between log consumption and log hours is flat, or slightly decreasing, depending on the definition of consumption.

**Micro facts:** The variance of changes in log wages declines weakly over the life-cycle, while the variance of changes in log hours exhibits a stronger decline. The correlation between changes in log wages and log hours is negative and does not vary systematically with age.

### 2.3 Time-series facts

**Macro facts:** The variance of log wages increased from 0.25 to 0.35 from 1967 to 1996. This 10 log-point increase is mostly concentrated in the period 1978-1992. The rise in the variance of earnings, from 0.28 to 0.42, is larger by roughly 4 log points. Behind this more rapid increase lies a substantial rise in the wage-hour correlation, whereas there is no noticeable change in the variance of hours. CEX data on consumption are only available in a consistent way since the 1980 survey. The variance of log consumption has risen since that date, but by a much smaller amount relative to the variance of earnings: 3.5 log points for non-durable consumption, and 5 points for durable consumption. These numbers are roughly 3 times smaller than the observed surge in the variance of log-individual earnings. This fact has been previously documented and discussed by Krueger and Perri (2006), and Attanasio, Battistin and Ichimura (2006).

The CEX data also show a remarkable rise in the wage-consumption correlation by 10 log points for non-durable consumption, while the hours-consumption correlation rises over the period 1980-1985, when the bulk of inequality occurred, and decreases after 1985.

**Micro facts:** The variance of changes in log wages shows a slow but continuous rise over the entire sample period, with a peak in the early 1990s. The variance of changes in log hours is quite stable, while the correlation between changes in wages and changes in hours rises substantially (15 log points), especially in the 1980s, the period of the sharp rise in wage dispersion.6

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6Moments involving consumption in first-differences are not available due to the lack of a reliable
3 Model economy

Demographics We adopt the perpetual youth, overlapping-generations framework developed by Yaari (1965). Agents are born at age zero and survive from age \( a \) to age \( a + 1 \) with constant probability \( \delta < 1 \). A new generation with mass \( (1 - \delta) \) enters the economy each period. Thus the measure of agents of age \( a \) is \((1 - \delta)\delta^a\).

Preferences Lifetime utility for an agent of age \( a \) born (i.e., entered in the labor market) in year \( b \) is given by

\[
\sum_{t=b}^{\infty} (\beta \delta)^{t-b} u(c_t, h_t, \zeta_t, \varphi),
\]

where period utility is

\[
u(c_t, h_t, \zeta_t, \varphi) = \frac{c_t^{\frac{1}{1-\gamma} - 1}}{1-\gamma} - \exp\left((\gamma + \sigma) \varphi + \sigma \zeta_t\right) \frac{h_t^{1+\sigma}}{1+\sigma}.
\]

Here \( c_t \) denotes consumption, and \( h_t \) is hours worked. The parameters \( \varphi \) and \( \zeta_t \) are random individual-specific preference weights that capture the strength of aversion to work (or taste for leisure) relative to the preference for consumption. We will assume that \( \varphi \) is a permanent effect known at the time of labor market entrance, while \( \zeta_t \) is a transitory shock drawn independently at each age.

Utility after death is normalized to zero, and agents discount at rate \( \beta \delta \), where \( \beta < 1 \) is the pure discount factor, and \( \delta \) is the survival rate. The parameter \( \gamma \) represents the coefficient of relative risk-aversion (\( 1/\gamma \) is the intertemporal elasticity of substitution for consumption). The Frisch elasticity of labor supply is equal to \( 1/\sigma \).

Separability in preferences between consumption and hours worked is a common assumption in the micro literature that estimates elasticities for consumption and labor supply (for a survey, see Browning, Hansen and Heckman, 1999). A popular alternative that is more common in the macro literature is to assume consumption and leisure are aggregated in a Cobb-Douglas fashion. In Heathcote, Storesletten and Violante (2005b) we show that it is also possible to solve the model in closed form in that case. Here we adopt the separable specification, primarily because it affords valuable flexibility in distinguishing between agent’s willingness to substitute consumption and hours inter-temporally.
We include preference variation because a large share of hours variation occurs at fixed wage rates (Abowd and Card, 1989). The fixed effect \( \varphi \) will capture permanent idiosyncratic differences in productivity in the home sector (perhaps related to differences in family composition) in addition to a pure permanent taste heterogeneity. The shock \( \zeta_t \) will capture transitory changes in the relative taste for leisure versus consumption, relating to events like bouts of ill health, or the timing of family celebrations or vacations which are largely predictable. We specify the distributions for \( \varphi \) and \( \zeta_t \) below.

**Production and individual labor productivity** Production takes place through a constant-returns-to-scale aggregate production function with labor as the only input. The labor market and the goods market are perfectly competitive, so individual wages equal individual productivities (units of effective labor per hour worked). The process for efficiency units \( w_t \) is given (in logs) by the sum of two orthogonal components capturing idiosyncratic shocks to wages:

\[
\log (w_t) = \alpha_t + \varepsilon_t
\] (3)

The crucial feature about the components \( \alpha_t \) and \( \varepsilon_t \) is that \( \alpha_t \) captures shocks that remain uninsured in equilibrium, while \( \varepsilon_t \) captures shocks that can be insured by individuals. In the next section we will describe the asset market structure that motivates our labelling of \( \alpha_t \) as the idiosyncratic uninsurable component of the wage, and \( \varepsilon_t \) as the insurable component.\(^7\)

We assume that uninsurable shocks are permanent in nature. Thus, the uninsurable component \( \alpha_t \) follows a random walk with normally distributed innovations:

\[
\alpha_t = \alpha_{t-1} + \omega_t \quad a \geq 1
\]

\[
\omega_t \sim N \left( \frac{-\nu_{\omega t}}{2}, \nu_{\omega t} \right).
\]

Note that the variance of the innovation \( \omega_t \) can be time-varying.

The insurable shocks \( \varepsilon_t \) may be permanent or transitory in nature. Thus, we model \( \varepsilon_t \) as a sum of two components:

\[
\varepsilon_t = \kappa_t + \theta_t,
\]

\(^7\) All results in the paper would be unchanged if we were to assume an aggregate shock \( z_t \) in addition to the idiosyncratic shocks, i.e. \( \log (w_t) = z_t + \alpha_t + \varepsilon_t \). As will be clear later, the process for \( z_t \) would play no role in determining second moments, so for the sake of simplicity we have abstracted from aggregate risk in the exposition of the model.
where $\theta_t$ is transitory and the permanent component $\kappa_t$ follows the unit root process

$$\kappa_t = \kappa_{t-1} + \eta_t \quad a \geq 1.$$ 

The innovation to the permanent component $\eta_t$ and the transitory component $\theta_t$ are both normally distributed with potentially time-varying variances,

$$\eta_t \sim N \left( -\frac{v_{\eta t}}{2}, v_{\eta t} \right) \quad a \geq 1$$

$$\theta_t \sim N \left( -\frac{v_{\theta t}}{2}, v_{\theta t} \right) \quad a \geq 0.$$

Upon entering the labor market at age 0 agents draw their initial idiosyncratic realization of these two components, $\alpha_0$ and $\kappa_0$, the initial transitory wage shock $\theta_t$, and idiosyncratic preference parameters $\varphi$ and $\zeta_t$. The initial draw for $\kappa_0$ captures permanent insurable shocks cumulated prior to the reaching the youngest age in our sample, and it is drawn from a normal distribution

$$\kappa_0 \sim N \left( -\frac{v_{\kappa 0}}{2}, v_{\kappa 0} \right).$$

The initial draw of $\alpha_0$ can be thought of as “innate ability”, or initial human capital obtained through schooling. The fixed effects $(\alpha_0, \varphi)$ are jointly drawn from a bivariate normal distribution:

$$\begin{pmatrix} \alpha_0 \\ \varphi \end{pmatrix} \sim N \left( \begin{pmatrix} -v_{\alpha 0}/(\gamma + \sigma) \\ v_{\alpha 0} \\ v_{\varphi} \end{pmatrix} \right), \begin{pmatrix} v_{\alpha 0} & v_{\varphi 0} \\ v_{\varphi 0} & v_{\varphi} \end{pmatrix}$$

where $e^\varphi = E[\exp ((\gamma + \sigma) \varphi)]$ is the mean population weight on the hours component of the utility function, $v_{\alpha 0}$ is the variance for the fixed effect $\alpha_0$, and $v_{\varphi}$ is the constant population variance for the preference weight $\varphi$. Moreover, $\varphi$ and $\alpha_0$ are allowed to be correlated with constant covariance $v_{\varphi \alpha}$.

Finally, the insurable preference shocks are drawn from a time-invariant distribution:

$$\zeta_t \sim N \left( -\frac{\sigma v_{\zeta}}{2}, v_{\zeta} \right),$$

so the average transitory preference weight is unity: $E \exp (\sigma \zeta) = 1$.

\*\*If human capital accumulation were endogenous, agents with low $\varphi$ would be more prone to accumulate since they would be willing to work more hours and hence face a higher return on their investment. They would therefore start their working life with a higher $\alpha_0$. Human capital theory thus seems to suggest a negative value for $v_{\varphi \alpha}$.\*\*
Given all the distributional assumptions above, the unconditional expected wage rate per efficiency unit is constant, and equal to 1. However, the average wage per hour actually worked will not equal one, since individual labor supply in general will depend on the individual wage. Note that all time variation in the idiosyncratic component of the process for wages is captured through the time-varying variances $v_{\omega t}$, $v_{\eta t}$, and $v_{\theta t}$. The process for preference shocks is time-invariant.

The process for wages described above is quite rich, and is potentially consistent with both the key features of individual wage dynamics as well as with the broad trends in wage dispersion across the life-cycle and through time. For example, the autocovariance function for individual wages in the PSID declines at a one period lag, indicating the presence of a transitory component in wages. At the same time cross-sectional wage dispersion increases approximately linearly with age, suggesting the presence of permanent shocks.\(^9\) In the empirical labor literature, the process for wages or earnings is often specified as combination of a unit root component and an MA(1) component (see, for example, Meghir and Pistaferri, 2004). However, the estimated coefficient on the lagged error component is typically small, and it is not always easy to reject the possibility that it is zero.\(^10\) In the interests of parsimony we therefore abstract from a moving average component.

Note that all dynamics in the wage-generating process are modelled as time effects, i.e., we assume no cohort effects. In Heathcote, Storesletten and Violante (2005a) we argue that time effects are required to account for the observed trends in inequality in thirty years of U.S. data, while there is little evidence that cohort effects have been important.\(^11\)

We assume that agents have perfect foresight regarding the future paths for $v_{\omega t}$, $v_{\eta t}$, and $v_{\theta t}$. This assumption is not required for tractability. An alternative assumption, which we plan to pursue in future work, is that the variances $v_{\omega t}$ and $v_{\eta t}$ themselves

\(^9\)Alternatively this pattern could be rationalized by transitory shocks whose variance increases with age. We reject this alternative interpretation because it would counter-factually imply a strong negative covariance between the change in individual wages between successive periods, $\text{cov}(\Delta w_t, \Delta w_{t+1})$.

\(^10\)For college graduates, Meghir and Pistaferri cannot reject the possibility that the number of moving average lags ($q$) is equal to zero. Similarly, Abowd and Card (1989) find no evidence of a moving average component in bi-annual National Longitudinal Survey of Men 45-59 data.

\(^11\)One might, however, suspect that cohort effects should play some role in the distribution of fixed effects. For example, rising college enrollment rates may have changed the level of permanent wage dispersion in younger relative to older cohorts. Later, we generalize the model to allow for cohort effects in the the variance of the initial draws of $\alpha$ and $\kappa$. 

13
follow known stochastic processes.\textsuperscript{12}

As traditionally done in this literature, in our model the evolution of efficiency units taken as exogenous. However, earnings are endogenous because of the flexible labor supply decision, so labor income is controlled by individuals to some extent. Huggett, Ventura and Yaron (2006a, 2006b) take a different approach: by building on the prototypical Ben-Porath model, they allow individuals to allocate time between work and investment in a risky human capital accumulation technology. In their model, individuals have exogenous labor supply but, through these investments they partially control the life-cycle evolution of efficiency units of labor.

**Sources of Insurance** To complete the description of our economy, we resort to the commonly used “island fiction”. Agents in our economy are born into a “group”, or on an “island”. New agents are born with zero financial wealth. Some of the risk agents face is purely idiosyncratic, whereas other risks are at the island-level. In particular, \((\varphi, \omega_t)\) are common to all agents within an island – this pair indexes an island – whereas each individual experiences idiosyncratic shocks \((\eta_t, \theta_t, \zeta_t)\).

The only assets traded between islands are risk-free non-contingent bonds. Within each island, perfect insurance arrangements exist that allow agents to fully share idiosyncratic risk. There are no durable goods in the economy; thus all traded assets are in zero net supply.

We are deliberately agnostic about what precisely constitutes an island/group, and about what mechanisms deliver perfect risk-sharing within an island. Rather, we present several alternative interpretations. For example, one could think of islands as families, firms, unions, countries, or other social groupings that can pool member-specific risks, but not shocks at the group level. Similarly, there are various different mechanisms that could deliver within-group risk-sharing. We will discuss three of them: (i) a group planner (for example, the head of household in the group-as-family interpretation) who simply dictates allocations, (ii) a government with access to lump-sum taxes and transfers, and (iii) financial markets in which agents can trade a complete set of state-contingent claims, period by period. In each case, the risk-free bond market is the only avenue for trade between islands.

\textsuperscript{12}These processes could potentially be functions of an aggregate productivity shock. This would allow for counter-cyclical variance of idiosyncratic risk.
In order to find a competitive equilibrium in heterogeneous-agents economies with incomplete markets, one must in general result to numerical techniques. However, in our economy, the careful chosen specifications for preferences, risks and opportunities for trade imply that there is an equilibrium in which there is no trade in non-contingent bonds. This no-trade result allows us to characterize allocations analytically.

### 3.1 Agents’ problem

We now describe an agent’s problem and define an equilibrium when risk is shared within islands via financial markets.\(^{13}\) Let \(s_t \equiv (d_t, \varepsilon_t, \zeta_t)\) denote the individual state, where \(d_t\) is individual wealth after assets have paid out in the current period. Let \(i_t \equiv (\alpha_t, \varphi)\) denote the island/group state. The aggregate economy-wide state is, in principle, the cross-sectional distribution of agents \(\Phi_t(ds, di)\). Let \(x_t \equiv (s_t, i_t, \Phi_t)\) denote the entire set of state variables potentially relevant for individual choices.

In each period \(t\) agents on an island trade a complete set of state-contingent insurance assets, one for each possible combination of shocks in the next period, i.e. these one period Arrow securities are contingent on \(\lambda_{t+1} \equiv (\omega_{t+1}, \eta_{t+1}, \theta_{t+1}, \zeta_{t+1})\). Let \(B_t(\lambda_{t+1}; x_t)\) and \(Q_t(\lambda_{t+1}; x_t)\) denote the quantity and the price of an insurance contract purchased at date \(t\) by an individual with state \(x_t\), paying off one unit of consumption, the numeraire good, if and only if the shock at \(t + 1\) is \(\lambda_{t+1}\). Let \(\phi_t(\lambda_{t+1})\) denote the probability of drawing \(\lambda_{t+1}\) next period.\(^{14}\)

Riskless one-period bonds are also traded. These bonds can be traded between any agents in the economy, i.e., also across islands/groups. Let \(b_t(x_t)\) and \(q_t(x_t)\) be the price and quantity of the riskless bond purchased by an individual with state \(x_t\).

An agent’s with state vector \(x_t\) faces a budget constraint at any age \(a_t > 0\) which can be written as

\[
c_t(x_t) + \int_{\Lambda_{t+1}} Q_t(\lambda_{t+1}; x_t)B_t(\lambda_{t+1}; x_t)d\lambda_{t+1} + q_t(x_t)b_t(x_t) = w_t(x_t)h_t(x_t) + d_t(x_t),
\]

and next period wealth is

\[
d_{t+1}(x_{t+1}) = B_t(\lambda_{t+1}; x_t) + b_t(x_t),
\]

\(^{13}\)In the Appendix we describe the two other allocation mechanisms described above: benevolent island planner, and benevolent government with access to taxes and transfers.

\(^{14}\)Note that the conditional probability distribution at date \(t\) over \(\lambda_{t+1}\) depends only on \(t\), and not on \(x_t\).
where \( x_{t+1} \) is consistent with \( x_t \) and \( \lambda_{t+1} \).

The initial conditions \( \alpha_0 \) and \( \varphi \) (and thus the location of the home island) are known before initial asset trade. Thus the initial vector of possible shocks against which agents may trade state-contingent claims is similar to that at any future date, except that \( \omega_t \) does not appear (the initial realization for \( \omega_t \) is drawn at age one), and \( \eta_t \) is replaced by \( \kappa_0 \). Let \( \lambda_0^o \equiv (\kappa_0, \theta_t, \zeta_t) \) denote the initial vector of shocks. Let \( \phi_t(\lambda_0^o) \) denote the date \( t \) probability of drawing \( \lambda_0^o \).

The initial budget constraint at age 0 is therefore
\[
\int Q_0^t(\lambda_0^o; i_t, z_t)B_0^t(\lambda_0^o; i_t)\,d\lambda_0^o = d_0^o = 0
\]
where \( d_0^o \) is the initial wealth endowment, assumed to be zero.

In addition to these budget constraints, agents face limits on borrowing that rule out Ponzi schemes, and the usual non-negativity constraints on consumption and hours worked.

### 3.2 Competitive equilibrium

A competitive equilibrium for this economy is a set of allocations \( \{c_t(x_t), h_t(x_t), b_t(x_t), B_t(\lambda_{t+1}; x_t), \lambda_t^0; i_t)\} \), prices \( \{q_t(x_t), Q_t(\lambda_{t+1}; x_t), \lambda_t^0; i_t)\} \), and probabilities \( \{\phi_t(\lambda_{t+1}), \phi_t(\lambda_0^o)\} \) for all \( t \), all \( x_t \) and all \( \lambda_{t+1} \) such that: 1) allocations maximize expected lifetime utility for the agents, taking as given initial wealth \( d_0^o = 0 \), probabilities \( \{\phi_t, \phi_t^0\} \) and the asset prices; 2) insurance markets clear island-by-island; 3) the economy-wide market for non-contingent bonds clears; 4) the probabilities \( \{\phi_t, \phi_t^0\} \) are consistent with the assumed time-varying distributions from which wage and preference shocks are drawn.

**PROPOSITION 1 (existence):** There exists a sequential competitive equilibrium characterized as follows:

- Bond purchases are zero at every state, while purchases of Arrow securities in any future state \( \lambda_{t+1} \) are equal to the difference between the expected present value of consumption and the expected present value of earnings, given the individual state \( (\varepsilon_{t+1}, \zeta_{t+1}, \alpha_{t+1}, \varphi) \) implied by \( (\varepsilon_t, \zeta_t, \alpha_t, \varphi) \) followed by \( \lambda_{t+1} \).

- The risk-free rate \( R_t = 1/q_t \) is given by
\[
R_t = \beta^{-1} \exp \left( \frac{1 + \sigma}{\sigma + \gamma} \left( \frac{\Delta v_t}{2\sigma} - \left( \frac{1 + \sigma}{\sigma + \gamma} + 1 \right) \frac{v_{\omega,t+1}}{2} \right) \right), \tag{4}
\]
and prices of Arrow securities are

\[
Q_t^0(\lambda^0_t; z_t) = \phi_t(\lambda^0_t)
\]
\[
Q_t(\lambda_{t+1}) = \phi_t(\lambda_{t+1}) \beta \exp \left( -\gamma \frac{1 + \sigma}{\sigma + \gamma} \left( \frac{\Delta v_{et}}{2\sigma} + \omega_{t+1} \right) \right),
\]

where \(\Delta v_{et} = \Delta v_{\theta t} + v_{n,t+1}\).

- Consumption and hours worked are given by

\[
c_t(\varepsilon_t, \zeta_t, \alpha_t, \varphi) = c_t(\alpha_t, \varphi) = \exp \left[ -\varphi + \frac{1}{\sigma + \gamma} \alpha_t + \frac{\sigma}{2 \gamma + \sigma} \left( \frac{v_{et}}{\sigma^2} + \zeta_t \right) \right] \tag{5}
\]
\[
h_t(\varepsilon_t, \zeta_t, \alpha_t, \varphi) = \exp \left[ -\varphi - \zeta_t + \frac{1}{\sigma + \gamma} \alpha_t + \frac{1}{\sigma} \varepsilon_t - \frac{\gamma}{2 \gamma + \sigma} \left( \frac{v_{et}}{\sigma^2} + \zeta_t \right) \right], \tag{6}
\]

where \(v_{et}\) is the cross-sectional variance of the insurable component of the wage.

- Note that allocations do not depend on wealth \(d_t\).

**Proof:** See Appendix.

**Tractability** We begin by discussing the key property of equilibrium: wealth does not appear as a state variable in equilibrium individual allocations, and the distribution of wealth does not appear in the expressions for equilibrium prices. In particular, in our framework the vector of shocks and initial heterogeneity \((\varepsilon_t, \zeta_t, \alpha_t, \varphi)\) is a sufficient statistic for individual choices, which is extremely convenient, since all these state variables are either constant (permanent preference heterogeneity), or evolve stochastically according to simple known exogenous processes. Analogously, prices are defined by simple functions of the known parameters defining the cross-sectional distribution over these individual states. The reason allocations and prices can be characterized without reference to the wealth distribution is twofold

First, within a particular island markets are effectively complete, and we can imagine an island-planner dictating allocations. Thus, within-island allocations can be characterized without reference to the within-island wealth distribution.\(^{15}\)

\(^{15}\)This first part of our result is reminiscent of some existing contributions. With complete markets, CRRA preferences, and heterogeneity in initial wealth endowments, Chatterjee (1994) shows that Gorman-aggregation holds within the neoclassical growth model: individual savings are linear in individual wealth, so average wealth is a sufficient statistics for aggregate dynamics. Maliar and Maliar (2003) generalize this result to the case of fully insurable productivity shocks. In these complete markets economies, the dynamics of the wealth distribution are easy to track.
Second, the reason the inter-island wealth distribution does not show up in allocations is that, given the permanent nature of island-level shocks, and the fact that all islands start out with zero holdings of non-contingent bonds, the cross-island distribution of debt remains degenerate. This result, which extends the no-trade result by Constantinides and Duffie (1996), relies critically on agents in different islands facing the same process for the expected stochastic discount factor absent trade in the non-contingent bond. This requirement is satisfied when (i) island-specific wage shocks are multiplicative and permanent (so that wage growth has the same mean and variance across islands), (ii) preferences over consumption are in the constant relative risk aversion class (so that high and low income agents have the same attitude to risk), (iii) islands all start out with zero aggregate wealth (so that the aggregate ratio of risky wage income to riskless bond income is equal across islands), and (iv) the process for idiosyncratic wages is such that island-level earnings growth is equal across islands. This last requirement reflects the fact that, in contrast to Constantinides and Duffie, hours are endogenous in an economy, so the processes for island-level earnings growth and wage growth are not necessarily the same.

Our result represents a substantial generalization of Constantinides-Duffie (1996): we extend it to an economy with a component of productivity shocks that is insurable, where agents supply labor elastically, are ex-ante heterogeneous in initial productivity endowments and taste, and face insurable taste shocks.\(^{16}\) Enriching the environment along these lines is paramount to confront the vast amount of heterogeneity present in micro data on consumption and labor supply.

Finally, note that while equilibrium allocations and prices can be characterized without reference to individual wealth \(d_t\) or the cross-sectional distribution of wealth, this result should not be interpreted as implying that wealth is always zero, or that decisions do not, in principle, depend on wealth. First, in equilibrium individual wealth \(d_t\) in the particular (complete) market structure described below is typically non-zero after time zero. Second, individual decisions do in principle depend on wealth, in the sense that two

---

\(^{16}\)To our knowledge, it is possible to solve for the equilibrium of these type of economies analytically only in two other cases. Benabou (2002) starts from the polar opposite assumption of autarky. In his economy, each agent lives on an island in isolation from the rest of the economy. With CRRA utility, given an assumption of i.i.d. log-normal productivity shocks, Benabou shows that the cross-sectional distribution of wealth has a closed form. The equilibrium can also be characterized analytically when preferences are in the CARA class (see, e.g., Wang, 2002).
otherwise identical agents with different amounts of wealth will make different choices. However, because each agent begins with zero wealth, agents that are otherwise identical at any point in time also have identical asset holdings in equilibrium, and these asset holdings are a particular known function of their current preference and wage shocks.

**Consumption and hours** Individual consumption is independent of the realization of the transitory shock $\varepsilon$, since that can be fully insured, but is re-scaled by the effect that the permanent shock $\alpha$ and the preference heterogeneity parameter $\varphi$ have on earnings. This consumption allocation is not what the simplest version of the PIH would imply. Consumption is still a random walk, as in Hall (1978), but there some permanent shocks are fully insurable, and thus do not affect consumption. In other words, our consumption allocation is consistent with the “excess smoothness” puzzle that recently has motivated a large amount of research to develop models that lie in between the bond-economy and complete markets (e.g., Attanasio and Pavoni, 2006). For $v_{\omega t} = 0$ our economy converges to complete markets, while for $v_{\alpha t} = v_{\gamma t} = 0$ it converges to the standard Imrohoroglu-Aiyagari-Huggett incomplete market model. In general, our model is an intermediate case, i.e., it is an economy with *partial insurance*.\(^{17}\)

Hours worked are increasing in the transitory shock $\varepsilon$, proportionately to the Frisch elasticity, since these shocks have a substitution effect but no income effect given that they are perfectly insured. Permanent shocks do have an income effect, and hours increase with $\alpha$ if and only if $\gamma < 1$. Hours decrease in the variance of the insurable shocks $v_{\varepsilon t}$ since the latter increases productivity and expected earnings, thus consumption (see Heathcote, Storesletten, and Violante, 2005b). The strength of this income effect on labor supply is mediated by $\gamma$.

**Risk-free rate** In a stationary world where $\Delta v_{\varepsilon t} = 0$, if $v_{\omega t} = 0$, the equilibrium interest rate would simply guarantee that the intertemporal saving motive is zero, i.e. $\beta R_t = 1$. In this case, there is no precautionary saving motive: the only risk is associated to the initial endowments of the fixed effects ($\varphi, \alpha_0$) which are drawn before markets open and any consumption/labor supply decision is made.

In general, the term on the RHS of (4) is strictly less than one whenever $v_{\omega} > 0$

\(^{17}\)We borrow this term from Blundell, Pistaferri and Preston (2005) in the hope that it will become the standard way to define this large class of economies which offer more risk sharing than a bond-economy but still less than full insurance.
and represents a precautionary saving motive that exactly counterbalances a (negative) intertemporal saving motive ($\beta R < 1$). It is easy to verify that the precautionary term is increasing in $\gamma$, the coefficient of risk aversion.\(^{18}\) The effect of $\sigma$ depends on the value of $\gamma$. If $\gamma > 1$, then the income effect on labor supply is stronger than the substitution effect and hours respond negatively to permanent shocks, see equation (??). In other words, labor supply is used as an insurance device. In this case, a higher Frisch elasticity of labor supply (lower $\sigma$) reduces the precautionary saving motive, since labor supply provides an hedge against risk. An interesting case is obtained by setting $\gamma = 1$ (the log-consumption case). If we let $\rho = 1/\beta - 1$, then we obtain that $\rho - r \simeq v_\omega$, so the precautionary saving motive exactly equals the size of the variance of the innovation to the uninsurable component of productivity.

Finally, $R_t$ is rising in the growth rate of the variance of the insurable risk, the term $\Delta v_{\epsilon t}$. Since consumption is increasing in the amount of insurable risk, a large growth in this risk signals large consumption growth, which in turn lowers the demand and the price of safe bonds.

### 4 Equilibrium cross-sectional moments

The first step in making the model operational is recognizing that measurement error is pervasive in micro data. We assume that consumption, earnings and hours worked are observed with measurement error, and that this measurement error is classical, i.e. i.i.d. over time and across agents. Let the measurement error in a variable $x$ be denoted $\mu^x$, with mean zero and variance $v_x$, and let $\hat{x} = x + \mu^x$ denote the empirical observation of $x$ including measurement error. Recall that we observe directly consumption, hours, and earnings, and compute hourly wages as earnings divided by hours.

The observed log allocations for individual $i$ are then given by

\[
\log \hat{w}_{it} = \alpha_{it} + \kappa_{it} + \theta_{it} + \mu^y_{it} - \mu^h_{it} \tag{7}
\]

\[
\log \hat{c}_{it} = -\phi_i + \frac{1 + \sigma}{\sigma + \gamma} \alpha_{it} + D^c_t + \mu_c^\epsilon \tag{8}
\]

\[
\log \hat{h}_{it} = -\varphi_i - \zeta_{it} + \frac{1}{\sigma + \gamma} \alpha_{it} + \frac{1}{\sigma} (\kappa_{it} + \theta_{it}) + D^h_t + \mu^h_{it} \tag{9}
\]

\(^{18}\)In the inelastic labor supply case, $\sigma \to \infty$, then the precautionary term is exactly proportional to $\gamma(1 + \gamma)$ which is the coefficient of relative prudence.
The variables $D^c_t$ are dummy variables constant across agents of age $a$ in period $t$ and are given by

$$
D^c_t = \frac{\sigma}{2\gamma + \sigma} \left( \frac{v^a_{xt}}{\sigma^2} + v_{\zeta} \right),
$$

$$
D^h_t = -\frac{\gamma}{2\gamma + \sigma} \left( \frac{v^a_{xt}}{\sigma^2} + v_{\zeta} \right),
$$

thus they are filtered out from the data through the first-stage regression discussed in Section 2. So, without loss of generality, in what follows we ignore the terms $\{D^c_t, D^h_t\}$ and we drop the $i$ subscript in every variable.

Let $\Delta x_t \equiv x_t - x_{t-1}$ denote the change in a variable $x$. From the above observed allocations, the individual changes in observed allocations can be expressed as

$$
\Delta \log \hat{w}_t = \omega_t + \eta_t + \Delta \theta_t + \Delta \mu^y_t - \Delta \mu^h_t
$$

(10)

$$
\Delta \log \hat{c}_t = \frac{1 + \sigma}{\sigma + \gamma} \omega_t + \Delta \mu^c_t
$$

(11)

$$
\Delta \log \hat{h}_t = \frac{1 - \gamma}{\sigma + \gamma} \cdot \omega_t + \eta_t + \Delta \theta_t + \frac{\sigma}{\sigma - \sigma} - \Delta \zeta_t + \Delta \mu^h_t.
$$

(12)

### 4.1 Time-series moments

Given the allocations (7)–(9), we can easily derive closed-form expressions for the unconditional second moments (variances and covariances) of the joint equilibrium distribution of wages, earnings, hours and consumption, only as a function of preference and risk parameters.

**Macro moments** The cross-sectional moments involving wages and hours are, respectively,

$$
\text{var} \left( \log \hat{w}_t \right) = v_{\alpha t} + v_{\kappa t} + v_{\theta t} + v_{\mu y} + v_{\mu h}
$$

(13)

$$
\text{var} \left( \log \hat{h}_t \right) = v_{\varphi} + v_{\zeta} - \frac{2(1 - \gamma)}{\sigma + \gamma} v_{\alpha \varphi}
$$

$$
+ \left( \frac{1 - \gamma}{\sigma + \gamma} \right)^2 v_{\alpha t} + \frac{1}{\sigma^2} (v_{\kappa t} + v_{\theta t}) + v_{\mu h}
$$

(14)

$$
\text{var} \left( \log \hat{y}_t \right) = v_{\varphi} + v_{\zeta} + \left( \frac{1 + \sigma}{\sigma + \gamma} \right)^2 v_{\alpha t} + \left( \frac{1 + \sigma}{\sigma} \right)^2 (v_{\kappa t} + v_{\theta t}) + v_{\mu y}
$$

(15)

$$
\text{cov} \left( \log \hat{h}_t, \log \hat{w}_t \right) = \frac{1 - \gamma}{\sigma + \gamma} v_{\alpha t} + \frac{1}{\sigma} (v_{\kappa t} + v_{\theta t}) - v_{\mu h}
$$

(16)
The variance of measured wages is simply the sum of all the orthogonal productivity components, plus the variances of measurement error in earnings and hours.

The variance of hours has five different components. First, the larger is heterogeneity in the taste for leisure \( v_\phi \) and in preference shocks \( v_\zeta \), the higher is the cross-sectional variance in hours. Second, the effect of the covariance between taste for leisure and ability depends on the role of labor supply. Recall that if \( \gamma > 1 \), the income effect dominates the substitution effect in labor supply so inequality in hours worked in the population arises because high (low) ability individuals will work fewer (more) hours. If \( v_{\phi \alpha} < 0 \), then high-ability individuals, for pure taste reasons, like to work more hours than low-ability ones. Thus, the stronger this covariance is, in absolute value, the smaller the variance of hours will be. Third, the variance of the uninsurable shock translates into hours dispersion proportionately to the distance between \( \gamma \) and one. As \( \gamma \) approaches one, uninsurable shocks have no effect on hours. Fourth, the variance of the insurable shocks increases hours dispersion proportionately to the Frisch elasticity (squared). Finally, measurement error in hours contributes to the observed dispersion.

With respect to the variance of earnings, irrespective of the value for \( \gamma \), uninsurable risk \( v_{at} \) always increases earnings variability. If \( \gamma = 1 \), then the effect is one for one since hours are unaffected, if \( \gamma < 1 \) \((> 1) \) then hours amplify (dampen) the effect of permanent shocks on earnings dispersion.

The covariance between wages and hours has three components. Transitory insurable shocks tend to make hours comove positively with wages. Whether the variance of uninsurable shocks increases or decreases the covariance once again depends on the value for risk aversion \( \gamma \). Measurement error in hours reduces the observed covariance between wages (earnings divided by hours) and hours.

We now turn to the moments involving consumption.\(^ {19} \)

\[
\text{var} (\log \hat{c}_t) = v_\phi - \frac{2(1+\sigma)}{\sigma+\gamma} v_{\alpha \phi} + \left( \frac{1+\sigma}{\sigma+\gamma} \right)^2 v_{at} + v_{\mu c}. \quad (17)
\]

\[
\text{cov} (\log \hat{h}_t, \log \hat{c}_t) = v_\phi - \frac{(1+\sigma)(1-\gamma)}{\sigma+\gamma} v_{\alpha \phi} + \frac{(1-\gamma)(1+\sigma)}{(\sigma+\gamma)^2} v_{at}. \quad (18)
\]

\[
\text{cov} (\log \hat{h}_t, \log \hat{w}_t) = \frac{1+\sigma}{\sigma+\gamma} v_{at}. \quad (19)
\]

\(^ {19}\)The covariances involving earnings can be obtained easily as linear combinations of the three covariances below.
Comparing the variance of consumption to the variance of earnings, the terms involving insurable preference and productivity shocks that appear in the expression for the variance of earnings do not show up in the variance of consumption. The logic is that with separable preferences, insurable shocks do not affect consumption dispersion. The covariance between hours and consumption is increasing in the degree of preference heterogeneity, while the effect of uninsurable risk depends on the value of $\gamma$. A negative correlation between the ability-component of productivity and taste for leisure ($v_{\varphi_\alpha} < 0$) in general increases the covariance.\footnote{The covariance may be reduced in this case, only in the presence of very strong income effects, i.e., $\gamma \gg 1$, when the reduction in hours from a permanent uninsurable shock is so large to reduce earnings, thus consumption.} Recall that, in the data, the covariance between wages and hours is negative for most of the sample, while the covariance between hours and consumption is positive. If $\sigma > \gamma > 1$, as one might expect, this combination can only be attained if $v_{\varphi} \gg 0$.

The covariance between consumption and wages is unaffected by transitory insurable shocks, while a larger dispersion in uninsurable shocks always increases this covariance: the latter fluctuations in productivity affect both wages and consumption in the same direction. Finally, note that none of these covariances is affected by preference shocks.

**Micro moments** Since we do not exploit the (weak) panel dimension of CEX, we can only use covariances of changes in hours and wages. These moments are taken across agents between date $t$ and $t + 1$ and can be written as:

\begin{align*}
\text{var} \left( \Delta \log \hat{w}_t \right) &= v_{\omega t} + v_{\eta t} + v_{\eta t} + v_{\eta t-1} + 2v_{\mu y} + 2v_{\mu h} \quad (20) \\
\text{var} \left( \Delta \log \hat{h}_t \right) &= \left( \frac{1 - \gamma}{\sigma + \gamma} \right)^2 v_{\omega t} + \frac{1}{\sigma^2} (v_{\eta t} + v_{\eta t} + v_{\theta t-1}) + 2v_{\chi} + 2v_{\mu h} \quad (21) \\
\text{cov} \left( \Delta \log \hat{h}_t, \Delta \log \hat{w}_t \right) &= \frac{1 - \gamma}{\sigma + \gamma} v_{\omega t} + \frac{1}{\sigma} (v_{\eta t} + v_{\eta t} + v_{\theta t-1}) - 2v_{\mu h}. \quad (22)
\end{align*}

### 4.2 Life-cycle moments

Given the OLG structure of our economy, we can recover all the above moments conditional on age $a$. In particular, it is interesting to analyze the within-cohort change in the various dimensions of inequality as households age.

**Macro moments** We begin by studying the change in the variances. Let $\Delta \text{var}_t^a (\hat{x})$
be the within-cohort change (i.e., between age $a$ and age $a+1$, between $t$ and $t+1$) in the variance of the measured variable $x$. Then, we obtain

\[
\Delta \text{var}_t^a(\log \hat{w}) = v_{\omega t} + v_{\eta t} + v_{\theta t} - v_{\theta t-1}
\]

(23)

\[
\Delta \text{var}_t^a(\log \hat{h}) = \left(\frac{1 - \gamma}{\sigma + \gamma}\right)^2 v_{\omega t} + \frac{1}{\sigma^2} (v_{\eta t} + \Delta v_{\theta t})
\]

(24)

\[
\Delta \text{var}_t^a(\log \hat{y}) = \left(\frac{1 + \sigma}{\sigma + \gamma}\right)^2 v_{\omega t} + \left(\frac{1 + \sigma}{\sigma}\right)^2 (v_{\eta t} + \Delta v_{\theta t})
\]

(25)

\[
\Delta \text{cov}_{t}^a(\log \hat{h}_t, \log \hat{w}_t) = \frac{1 - \gamma}{\sigma + \gamma} v_{\omega t} + \frac{1}{\sigma} (v_{\eta t} + \Delta v_{\theta t})
\]

(26)

The rise in wage inequality over the life-cycle is determined by the variance of the innovations to the permanent insurable and uninsurable components, and by the change in the variance of the transitory insurable component. In the absence of changes in the transitory component (e.g., in a stationary world), the model suggests that the variance of hours should be increasing over the life cycle for the same reasons as wages, though with different weights on the variances of insurable and uninsurable permanent shocks. In the log-consumption case, only the former matters for hours. Note, in particular, that the model can generate a decline in the variance of hours over the life-cycle only through time-effects, i.e. because the variance in transitory insurable productivity falls ($\Delta v_{\theta t} < 0$).

A similar logic applies to the life-cycle pattern of the variance of earnings. Relative to the growth in the variance of wages, the growth in the variance of earnings will be larger the smaller is $\sigma$ and the smaller is $\gamma$.

Whether the covariance between wages and hours rises or falls over the life cycle depends on the value for risk aversion and the relative size of permanent and transitory innovations. When $\gamma > 1$, hours and the permanent component of wages covary negatively, thus the cumulation of permanent shocks pushes down the covariance as individuals age, but the cumulation of the transitory/insurable shocks pulls it up, in proportion to the Frisch elasticity $\frac{1}{\sigma}$. For $\gamma > 1$ and relatively large permanent shocks, the covariance between wages and hours could decrease with age.
Turning to the moments involving consumption over the life-cycle, we obtain:

\[
\Delta \text{var}_t (\log \hat{c}) = \left(\frac{1+\sigma}{\sigma+\gamma}\right)^2 v_{\omega t} \quad (27)
\]

\[
\Delta \text{cov}_t \left(\log \hat{h}_t, \log \hat{c}_t\right) = \frac{(1-\gamma)(1+\sigma)}{(\sigma+\gamma)^2} v_{\omega t} \quad (28)
\]

\[
\Delta \text{cov}_t (\log \hat{c}_t, \log \hat{w}_t) = \frac{1+\sigma}{\sigma+\gamma} v_{\omega t}. \quad (29)
\]

The change in the variance of consumption over the life-cycle is determined only by the size of the variance of the innovation to the uninsurable component, while the size of insurable shocks (both permanent and transitory) has no impact. The uninsurable-shock multiplier \(\left(\frac{1+\sigma}{\sigma+\gamma}\right)^2\) for earnings and consumption is exactly one either in the log case or in the case of inelastic labor supply, i.e. \(\sigma \to \infty\). In general, the model predicts that the growth in the variance of earnings over the life-cycle should be larger than that for consumption but, interestingly, it could be lower than the growth in the variance of wages, if \(\gamma\) is sufficiently larger than one, so that wages and hours comove very negatively.

Hours and consumption are related only through uninsurable shocks. When \(\gamma > 1\), hours move up in response to a negative shock, while consumption moves down, so that the covariance falls with age as permanent shocks grow in importance. The model predicts that the covariance between consumption and wages, by contrast, will increase over the life cycle, in proportion to the variance of uninsurable innovations. Finally, note that none of the changes in variances and covariances over the life-cycle are affected by preference heterogeneity/shocks or measurement error, as the variances of these are assumed constant.

**Micro moments** There is a close link between changes between \(t\) and \(t+1\) in the within-cohort cross-sectional (co)variances for wages and hours, and the (co)variance of individual changes in these same variables between \(t\) and \(t+1\).

Covariances of lagged individual changes at lag one can be obtained as a linear combination of the other moments. For example,

\[
cov_t (\Delta \log w_{-1}, \Delta \log w) = \frac{1}{2} \left[\Delta \text{var}_t (\log w) - \text{var}_t (\Delta \log w)\right],
\]

and so on. Hence, these moments do not add any new information relative to what already contained in the other cross-sectional moments. Covariances of the individual changes are all zero beyond lag one.
4.2.1 A comparison with Blundell-Preston (1998)

Blundell and Preston (1998) develop a clever theoretical framework to identify permanent and transitory income innovations from joint household data on consumption $c$ and income $y$. Their baseline model is that of a household with quadratic-utility, discount rate equal to the interest rate, and no liquidity constraint, i.e. the classical Hall (1978) model where consumption follows a random walk. They assume that labor income is the sum of two components, a permanent shock ($\alpha_t$ in our notation) and a transitory shock ($\theta_t$ in our notation).\footnote{They allow $\theta_t$ to be a MA(1) process, but this generalization is not crucial for their results and for the point we make below.} They show that, in their model, for a cohort of age $a$ at time $t$,

$$\Delta \text{var}_t^a (c) = \Delta \text{cov}_t^a (c, y) \simeq \omega \alpha$$ (30)

$$\Delta \text{var}_t^a (y) - \Delta \text{var}_t^a (c) \simeq \Delta \theta.$$ (31)

From equations (23) to (27), in our economy:

$$\Delta \text{var}_t^a (\log c) = \Delta \text{cov}_t^a (c, y) = \left(\frac{1 + \sigma}{\sigma + \gamma}\right)^2 \omega \alpha$$ (31)

$$\Delta \text{var}_t^a (\log \hat{y}) - \Delta \text{var}_t^a (\log c) = \left(\frac{1 + \sigma}{\sigma}\right)^2 (\eta \alpha + \Delta \theta).$$

A comparison of (30) and (31) reveals immediately one role of endogenous labor supply. The change in the variance of log consumption identifies correctly $\omega \alpha$ only if either $\gamma = 1$ or $\sigma \to \infty$. In the first case, income and substitution effect of an uninsurable shock on labor supply cancel out, thus the original permanent innovation to productivity translates one for one into consumption. In the second case, trivially, labor supply has no role.

The second equation provides a difference in difference estimator for the change in the variance of the transitory shock. Once again, our model suggests that the more elastic labor supply is, the larger will be the difference between the left hand side and the true change in the transitory productivity shock $\Delta \theta$. Intuitively, when agents’ labor supply is very sensitive to transitory insurable shocks, the response of labor income is magnified. This second equation also clarifies another issue: productivity shocks could contain a permanent component ($\kappa_t$ in our notation) that is insurable, and thus does not affect consumption. This possibility is not considered by Blundell and Preston.
5 Identification

Identification of the whole model requires two additional assumptions, namely that the variance of measurement error in either hours or earnings can be calibrated using external estimates, and knowledge of the consumption-hours correlation \( \text{corr} (\log c, \log h) \). These assumptions are formally stated as follows.

**Assumption A1 (Measurement Error)** (1) There exists external estimates of either \( v_{\mu h} \) or \( v_{\mu y} \). (2) There exists estimates of either \( v_{\mu c} \) or of the consumption-hours covariance, i.e. the level analogue of the moment in eq. (28).

We are now ready for the second main result of the paper, namely that the macro and micro moments for one single cohort can identify the parameters of the model, given that Assumption A1 holds. Moreover, if we have the process for measurement error in both hours and earnings, we can identify the model without any data on consumption allocations. We state this as a formal proposition. With a slight abuse of notation, we let the time-varying moments \( v_{xt} \) refer to the within-cohort variances in period \( t \) for a cohort which enters the economy in period zero.\(^{22}\)

**Proposition 2 (Identification)** (A) Suppose Part 1 of Assumption A1 holds. Then, time series of the macro and micro moments, without any of the covariances that involve consumption, identify the model. That is, they identify the parameters \( \{\sigma, \gamma, v_\zeta, v_{\mu h}, v_{\mu y}, v_{\alpha 0}, v_{\kappa 0}\} \), the sequences \( \{v_{\omega t}\}_{t=1}^{T-1} \) and \( \{v_{\omega t+1}, v_{\eta t}\}_{t=0}^{T-1} \), and the variance of the final-period insurable innovations \( v_{\eta T} + v_{\theta T} \). (B) If in addition Part 2 of Assumption A1 holds, then the parameters \( \{v_{\varphi}, v_{\alpha \varphi}, v_{\mu c}\} \) are identified.

**Proof** We start by proving Part A of the proposition.

1. Identify \( \sigma, \gamma, \{v_{\omega t}, \Delta v_{\omega t}\}_{t=1}^{T} \), where \( \Delta v_{\omega t} = v_{\eta t} + \Delta v_{\theta t} \). Consider the following changes

\(^{22}\)Therefore, in this section \( \{v_{xt}\}_{t=1}^{T} \) corresponds to \( \{v_{x,0+a}\}_{a=1}^{T} \) in our previous notation.
(over time) in the macro moments in eq. (13), (17), (14), and (20):

\[
\Delta \text{var} (\log \hat{w}_t) = v_{\omega t} + \Delta v_{\omega t}
\]
\[
\Delta \text{var} (\log \hat{c}_t) = \left(\frac{1 + \sigma}{\sigma + \gamma}\right)^2 v_{\omega t}
\]
\[
\Delta \text{var} (\log \hat{h}_t) = \left(\frac{1 - \gamma}{\sigma + \gamma}\right)^2 v_{\omega t} + \frac{\Delta v_{\omega t}}{\sigma^2}
\]
\[
\Delta \text{cov} (\log \hat{w}_t, \log \hat{h}_t) = \left(\frac{1 - \gamma}{\sigma + \gamma}\right) v_{\omega t} + \frac{\Delta v_{\omega t}}{\sigma}.
\]

Clearly, these moments yield four linearly independent equations identifying \(\sigma\), \(\gamma\), \(v_{\omega t}\), and \(\Delta v_{\omega t}\).

2. The sequences \(\{v_{\alpha t}, v_{\kappa t}\}_{t=0}^{T-1}\) and \(\{v_{\eta t}\}_{t=1}^{T-1}\) can be identified by rearranging eq. (13)-(16) and (20)-(22) to yield the following two linearly independent equations:

\[
\text{var} (\log \hat{w}_{t+1}) + \text{var} (\log \hat{w}_t) - \text{var} (\Delta \log \hat{w}_{t+1}) = 2v_{\alpha t} + 2v_{\kappa t},
\]
\[
\text{cov} (\log \hat{h}_{t+1}, \log \hat{w}_{t+1}) + \text{cov} (\log \hat{h}_t, \log \hat{w}_t) - \text{cov} (\Delta \log \hat{h}_{t+1}, \Delta \log \hat{w}_{t+1}) = 2 \left(\frac{1 - \gamma}{\sigma + \gamma}\right) v_{\alpha t} + \frac{2}{\sigma} v_{\kappa t}.
\]

The sequence \(\{v_{\kappa t}\}_{t=0}^{T-1}\) in turn determines \(\{v_{\eta t}\}_{t=1}^{T-1}\).

3. Identify \(\{v_{\theta t}\}_{t=0}^{T-1}, v_{\zeta}, v_{\mu y}, v_{\mu h}\)\). Suppose Part 1 of Assumption A1 is satisfied. Using a combination of the micro and macro moments in eq. (13)-(16) and (20)-(22) yields three linearly independent equation which, given knowledge of either \(v_{\mu h}\) or \(v_{\mu y}\), identifies \(\{v_{\theta t}\}_{t=0}^{T-1}, v_{\zeta}, v_{\mu y}, v_{\mu h}\)\):

\[
\frac{1}{2} (\text{var} (\Delta \log \hat{w}_{t+1}) - \Delta \text{var} (\log \hat{w}_{t+1})) = v_{\theta t} + v_{\mu y} + v_{\mu h}
\]
\[
\frac{1}{2} (\text{cov} (\Delta \log \hat{h}_{t+1}) - \Delta \text{var} (\log \hat{h}_{t+1})) = \frac{1}{\sigma^2} v_{\theta t} + v_{\zeta} + v_{\mu h}
\]
\[
\frac{1}{2} (\text{cov} (\Delta \log \hat{h}_{t+1}, \Delta \log \hat{w}_{t+1}) - \Delta \text{cov} (\log \hat{h}_{t+1}, \log \hat{w}_{t+1})) = \frac{v_{\theta t}}{\sigma} - v_{\mu h}.
\]
4. Identifying \( \{v_\varphi, v_{\alpha \varphi}, v_{\mu c}\} \). The equations

\[
\text{var}\left(\log \hat{c}_t\right) = v_\varphi - \left(\frac{1 + \sigma}{\sigma + \gamma}\right) v_{\alpha \varphi} + \left(\frac{1 + \sigma}{\sigma + \gamma}\right)^2 v_{\alpha t} + v_{\mu c}
\]

\[
= \frac{1}{2} \left(\text{var}\left(\log \hat{h}_{t+1}\right) + \text{var}\left(\log \hat{h}_t\right) - \text{var}\left(\Delta \log \hat{h}_{t+1}\right)\right)
\]

\[
= v_\varphi - \left(\frac{1 - \gamma}{\sigma + \gamma}\right) v_{\alpha \varphi} + \left(\frac{1 - \gamma}{\sigma + \gamma}\right)^2 v_{\alpha t} + \frac{1}{\sigma^2} v_{\kappa t},
\]

yield two restrictions on \( \{v_\varphi, v_{\alpha \varphi}, v_{\mu c}\} \). From Part 2 of Assumption A1, we get identification by either an external estimate of \( v_{\mu c} \) or by adding eq. (18) as a moment:

\[
\text{cov}\left(\log \hat{c}_t, \log \hat{h}_t\right) = v_\varphi - \left(\frac{1 + \sigma}{\sigma + \gamma}\right) v_{\alpha \varphi} + \left(\frac{1 - \gamma}{\sigma + \gamma}\right) \left(\frac{1 + \sigma}{\sigma + \gamma}\right)^2 v_{\alpha t}.
\]

5. Given an estimate of \( \Delta v_{\varepsilon T} \) and \( v_{\theta T - 1} \), it follows that the final-period sum \( v_{\eta T} + v_{\theta T} \) is identified as \( v_{\eta T} + v_{\theta T} = \Delta v_{\varepsilon T} + v_{\theta T - 1} \).

A number of comments are in order. First, as stated in Proposition 2, data over time for one single cohort is sufficient to identify the model. The reason is that our identification scheme exploits only within-cohort changes in inequality, precisely as in Blundell and Preston (1998). Following several cohorts over time therefore provides over-identifying restrictions and also allows identification of cohort effects in the initial draws of the uninsurable fixed effect \( v_{\alpha t} \) and in the initial insurable permanent component \( v_{\kappa t}^0 \).

Identification requires knowledge of the variance of measurement error in either hours or earnings. This is necessary in order to estimate the average level of the variance of the transitory insurable shocks \( v_{\theta t} \). However, the changes in these shocks, \( \Delta v_{\theta t} \), can be identified without any knowledge of measurement error. An alternative but less desirable assumption we could have imposed (to get identification) is that the variance of either \( v_{\theta t} \) or the permanent innovations \( v_{\eta t} \) is time invariant, or that we know the variance of the transitory preference shocks \( v_\zeta \).

Finally, identification of the model requires data on consumption (as stated in Proposition 2). Our PSID data on hours worked and earnings start in 1968, while our data on consumption do not start until 1980. Fortunately, this is not a problem for identification because we have at least two subsequent years with (cross-sectional) consumption data. We state this finding as a corollary.
Corollary Suppose there exists estimates of \( \text{var} (\log \hat{c}_t) \) for only the last part of the sample, i.e. from \( t = T^* > 0 \) to \( T \). If \( T^* < T \) and Assumption A1 holds, then the model is identified as in Proposition 2.

Proof Given Assumption A1, the elasticities \( \gamma \) and \( \sigma \) can be estimated using data from time \( t = T^* \) to \( t = T \). Given these estimates, \((32)-(34)\) identify \( \{v_{\omega t}, \Delta v_{\omega t}\}_{t=1}^{T} \) for the time period. The proof then proceeds as in the proof of Proposition 2.

6 Estimation method and results

To estimate the structural parameters of the model we use the Minimum Distance Estimator (MDE) introduced by Chamberlain (1984) where we minimize a weighted squared sum of the difference between each moment in the model and its data counterpart. The structural parameters to be estimated in the model are \( \{\gamma, \sigma, \upsilon_{\alpha\varphi}, \upsilon_{\xi}, \upsilon_{\alpha\theta}, \upsilon_{\mu y}, \upsilon_{\mu h}, \upsilon_{\mu c}\} \), and \( \{\upsilon_{\mu t}, \upsilon_{\omega t}, \upsilon_{\eta t}\}_{t=1}^{T_{\text{PSID}}} \). Given our discussion of identification above, we set \( \upsilon_{\mu h} \) exogenously, based on the estimates by Bound and Krueger (1991), and include the covariance between log consumption and log hours and the covariance between log consumption and log wages from the CEX. In this way, we estimate \( \upsilon_{\mu c} \) and \( \upsilon_{\mu y} \). Moreover, for the time being we have restricted \( \upsilon_{\alpha\varphi} = 0 \). In total, we have \( 3 \times T_{\text{PSID}} + 8 = 98 \) parameters to be estimated. Denote by \( \Theta \) this parameter vector.

The moments in the data are estimated from the relevant residuals of the first-stage regression using sample analogues of the corresponding population moments. We construct three types of moments: unconditional moments by 5-year age group, unconditional moments by year, and conditional moments by year and 5-year age group. In the benchmark estimation we use nondurable consumption. The moments in first-differences are only used in the time-series dimension, because as shown in the discussion of identification, these moments in the age dimension are linear combinations of the moments we already target. Let \( T_{\text{PSID}} = 30 \) and \( T_{\text{CEX}} = 18 \) be the number of years in the PSID and CEX sample, respectively, and \( A = 6 \) be the age range. Overall, when we include the covariances from CEX, we have 1,131 moments. We stack these moments in a long vector and denote any given moment by \( \hat{m}_j \) where the index \( j = 1, ..., J \) denotes the position in the vector. Let \( m(\Theta, j) \) be the corresponding theoretical moment.\(^{23}\)

\(^{23}\)Since the model’s period is one year, to construct the model’s moments by 5-year age brackets, we
Our MDE solves the following minimization problem

\[
\min_{\Theta} \left[ \hat{m} - m(\Theta) \right]' W \left[ \hat{m} - m(\Theta) \right],
\]

where \(\hat{m}\), and \(m(\Theta)\) are the \((J \times 1)\) vectors of the stacked empirical and theoretical covariances, and \(W\) is a \((J \times J)\) weighting matrix. Standard asymptotic theory implies that the estimator \(\hat{\Theta}\) is consistent, asymptotically Normal, and has asymptotic covariance matrix \(V = (D'WD)^{-1} D'W \Delta W D (D'WD)^{-1}\), where the matrix \(D \equiv \mathbb{E} [\partial m(\Theta) / \partial \Theta']\) and the matrix \(\Delta \equiv \mathbb{E} [(\hat{m} - m(\Theta)) (\hat{m} - m(\Theta))']\) are estimated via their empirical analogs to compute standard errors.

To implement the estimator, we need a choice for \(W\). The bulk of the literature follows Altonji and Segal (1996) who found that in common applications there is a substantial small sample bias in the estimates of \(\Theta\), hence using the identity matrix for \(W\) is a strategy superior to the use of the optimal weighting matrix characterized by Chamberlain (1984). With this choice, the solution of (38) reduces to a nonlinear least square problem.²⁴

### 6.1 Estimation results

We organize our findings in three parts. First, we discuss the parameter estimates. Second, we display the fit of the model over the life cycle and the time series. Here, we exploit the structural model to decompose the evolution of all our cross-sectional moments into the different sources of variation: preference heterogeneity/shocks, uninsurable and insurable productivity shocks, and measurement error. Given the estimated parameters, the additive and orthogonality structure of these four components in our moments makes the unique decomposition straightforward to compute by simply using our formulas. Finally, we perform a sensitivity analysis.

²⁴We deviate only slightly from Altonji and Segal. We normalize the weight of each age/time moment by one. Thus, we weight the unconditional moments by age with the number of years in the (PSID or CEX) sample, and the unconditional moments by time with the number of age brackets.

assume a constant yearly survival probability of \(1 - 1 / (55 - 25)\) and we aggregate across the five adjacent ages.
6.1.1 Parameter estimates

Table 2 reports parameter estimates and standard errors. For our two key preference parameters, we estimate $\gamma = 2.42$ and $\sigma = 5.94$. The implied intertemporal elasticity of substitution, 0.41, is within the standard range of existing estimates (for a survey, see Attanasio, 1999). The implied Frisch elasticity of labor supply is 0.17, a value that is consistent with the microeconomic evidence for males, which represent the vast majority of heads of households in PSID and CEX (for a survey, see Blundell and MaCurdy, 1999).

The initial variance of the insurable and uninsurable wage components $v_{\alpha 0}$ and $v_{\varepsilon 0}$ are rather similar in size and quite large, between 0.07 and 0.08, representing jointly 2/3 of the initial wage variation at age 25 (with the rest being explained by the iid insurable shocks). The conditional variance of the innovation to the uninsurable shock (Figure 7) is very close to zero for the first half of the sample, but it grows substantially in the 1980s and in the early 1990s. Its sample average, though, remains quite small, around 0.003. The variance of the permanent insurable shock starts growing slightly before the uninsurable one and remains high throughout the 1980s. Recall that the 1980s is the period when the bulk of the rise in inequality takes place. Its sample average is 0.005, substantially larger than the uninsurable shock. The variance of the iid insurable shock (Figure 8) is much larger than its permanent counterpart (by a factor of 7). It grows slowly throughout the sample and it peaks in the first half of the 1990s, consistent with the findings of Moffitt and Gottschalk (2002) from estimated earnings dynamics.

Table 2 also shows that measurement error in hours and consumption account for, respectively, 19% and 33% of the total cross-sectional variances for these variables. Thus, as expected, hours worked and consumption seem particularly plagued by measurement error. Moreover, according to our estimates, fixed preference heterogeneity is twice as large as insurable transitory preference shocks, and the latter shocks are, approximately, half as large as transitory insurable productivity innovations.

6.1.2 Life-cycle: fit and decomposition

Figures 8 and 9 demonstrate that the fit of the model over the life cycle is quite good. The model generates an increase in the variance of earnings that is slightly above the one...
Table 2: Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter description</th>
<th>Value (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$ Coefficient of risk aversion</td>
<td>2.42 (0.031)</td>
</tr>
<tr>
<td>$\sigma$ Inverse of Frisch elasticity</td>
<td>5.94 (0.140)</td>
</tr>
<tr>
<td>$\nu_{\alpha 0}$ Var. of initial uninsurable shock</td>
<td>0.08 (0.022)</td>
</tr>
<tr>
<td>$\nu_{\epsilon 0}$ Var. of initial insurable shock</td>
<td>0.071 (0.0005)</td>
</tr>
<tr>
<td>$\nu_{\omega}$ Avg. var. of uninsurable innovation</td>
<td>0.0032 (-)</td>
</tr>
<tr>
<td>$\nu_{\eta}$ Avg. var. of insurable permanent innovation</td>
<td>0.005 (-)</td>
</tr>
<tr>
<td>$\nu_{\theta}$ Var. of insurable transitory shock</td>
<td>0.036 (-)</td>
</tr>
<tr>
<td>$\nu_{\varphi}$ Var. of fixed preference heterogeneity</td>
<td>0.040 (0.0001)</td>
</tr>
<tr>
<td>$\nu_{\zeta}$ Avg. var. of insurable preference shock</td>
<td>0.018 (0.022)</td>
</tr>
<tr>
<td>$\nu_{\mu y}$ Var. of meas. error in earnings</td>
<td>0.002 (0.001)</td>
</tr>
<tr>
<td>$\nu_{\mu h}$ Var. of meas. error in hours</td>
<td>0.017 (-)</td>
</tr>
<tr>
<td>$\nu_{\mu c}$ Var. of meas. error in consumption</td>
<td>0.059 (0.001)</td>
</tr>
</tbody>
</table>

in the data, by 3 log points. The reason is twofold. On the one hand, the variance of hours in the model is slightly increasing, while in the data it has a mild downward trend. Equation (14) for the variance of hours in the model shows clearly that the model cannot generate a decreasing pattern for this moment. Over the life-cycle both insurable and uninsurable shocks cumulate, leading to more hours dispersion. Since $\gamma$ is quite close to one, and the Frisch elasticity is low, overall the model predicts a very weak rise.\textsuperscript{26} Second, the model predicts a mildly increasing correlation between hours and wages, while in the data this moment has a small downward trend. From equation (16) one can see the model can predict both positive and negative trends, depending on the relative size of insurable and uninsurable innovations to productivity, as well as on the value of $\gamma$. Since insurable

\textsuperscript{26}Within a self-insurance model, Kaplan (2006) can generate the observed decline in the variance of hours at the beginning of the life-cycle. This is due to (1) age effects in the transitory component of wages which are decreasing at the young ages; (2) a non-degenerate distribution of initial wealth which increases the variance of hours for young agents by lowering the covariance between wages and consumption.
innovations are slightly bigger, they induce a small positive trend.

The variance of consumption in the model grows as much as in the data over the life-cycle, in particular the model generates a growth in consumption dispersion that is three times smaller than earnings dispersion. This is due to the fact that, when comparing uninsurable and insurable shocks, the latter are relatively larger. The covariance between wages and hours grows in the model, while it falls slightly in the data. The reason, once again, is that insurable permanent shocks are estimated to be larger than uninsurable permanent innovations. The cumulation of these latter shocks over the life cycle explains why the model predicts, correctly, a rise in the consumption-wage correlation and slightly overpredicts the fall in the consumption-hour correlation over the life-cycle.

To decompose the life-cycle moments into preference heterogeneity/shocks, productivity shocks and measurement error, we set productivity shocks to their average value over the sample period and simulate, from the theoretical moments an artificial life-cycle profile. Figures 10 and 11 tell an interesting story. The rise in wages and earnings dispersion over the life cycle is explained by the cumulation of permanent productivity shocks that are, mostly, insurable. Preference heterogeneity and measurement error are the main determinant of the variance of hours worked, accounting for over 3/4 of its level, but productivity shocks entirely explain for its movements over the life cycle. Clearly, the rise in consumption dispersion is entirely due to uninsurable wage shocks, even though measurement error accounts for quite a big fraction of its level, around 1/3.

6.1.3 Time series: fit and decomposition

The model is able to predict a steep increase in earnings inequality vis-a-vis the relatively small rise in consumption dispersion and the flat profile of hours dispersion (Figure 12). The decompositions of Figure 13 show that since most of the observed rise in inequality over the period was insurable, this did not translate into a large rise in consumption dispersion. At the same time, a low Frisch elasticity is needed to explain why the variance of hours has not increased, in the wake of the surge in insurable productivity dispersion. Note that, even though we do not have data on consumption before 1980, we can still identify the variances of the uninsurable shock prior to that date (this information is embedded in the consumption dispersion of the older cohorts). Projecting backward, the model predicts a U shape in consumption inequality over the whole sample period, with
inequality declining in the early 1970s. Even though we do not have direct evidence of this, because of lack of data, this pattern is potentially consistent with the well known dynamics of the skill premium over the period.

The model also replicates the observed rise in the wage-hour and wage-consumption correlation (Figures 14). The decomposition in Figure 15 reveals that the rise in the variance of permanent uninsurable shocks induced a decline in the wage-hour correlation, while the rise in the variance of insurable shocks pushed this correlation upward. This second force dominated, especially in the 1980s. Insurable shocks have no role in determining the wage-consumption correlation. As clear from Figure 15, the movements in the latter are entirely due to the rise in the variance of uninsurable shocks.

The fall in the consumption-hour correlation is overestimated by the model. Once again, the rise in the dispersion of uninsurable productivity fluctuations coupled with \( \gamma > 1 \) leads to a negative income effect that pushes this correlation down in the model, slightly more than what called for by the data.

Moving now to the time-series representation of the micro facts, we note that the variance of changes in wages in nailed exactly by the model, since the dynamics of this moment exactly identifies the variance of iid insurable shocks. The model is also consistent with a stable variance of changes in hours: the low Frisch elasticity makes this moment quite unresponsive to the trends in insurable and uninsurable productivity dispersion.

6.1.4 Sensitivity analysis

The first of our robustness experiments is to use total consumption instead of nondurable consumption in the estimation. The parameter estimates, reported on the second line of Table 3, are very close to the benchmark estimates, so the precise definition of consumption used does not seem to affect the results.

In the second experiment, we relax the assumption that \( \{v_{o0}, v_{e0}\} \) are time-invariant and introduce cohort effects in these initial variances of the insurable and uninsurable shocks. Figure 16 shows that the estimated cohort effects fluctuate over time without too much persistence and tend to be negatively correlated. Overall, introducing cohort effects reduces substantially the estimate of the variance of transitory insurable productivity shocks, but it has little impact on the other parameters.

To understand the role of preference shocks and preference heterogeneity, we run two
Table 3: Parameter Estimates for Alternative Models

<table>
<thead>
<tr>
<th></th>
<th>$\sigma$</th>
<th>$\gamma$</th>
<th>$v_\omega$</th>
<th>$v_\eta$</th>
<th>$v_\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>5.94</td>
<td>2.42</td>
<td>0.0032</td>
<td>0.005</td>
<td>0.036</td>
</tr>
<tr>
<td>Total cons.</td>
<td>6.52</td>
<td>1.87</td>
<td>0.0032</td>
<td>0.005</td>
<td>0.036</td>
</tr>
<tr>
<td>Cohort effects</td>
<td>5.79</td>
<td>2.39</td>
<td>0.0029</td>
<td>0.005</td>
<td>0.025</td>
</tr>
<tr>
<td>No pref shocks</td>
<td>3.29</td>
<td>2.17</td>
<td>0.0034</td>
<td>0.005</td>
<td>0.036</td>
</tr>
<tr>
<td>No pref heterogeneity</td>
<td>2.02</td>
<td>2.32</td>
<td>0.0044</td>
<td>0.003</td>
<td>0.027</td>
</tr>
</tbody>
</table>

experiments where we shut down these channels, one at the time. When $v_\zeta = 0$, most of the macro moments are fitted equally well. Figure 18 plots some of the micro moments over the time series dimension. Also for this subset of moments the fit is quite good, except perhaps for the covariance between changes in wages and changes in hours which increases too much due to the higher Frisch elasticity—estimated now to be 0.30 instead of 0.17 as in the benchmark. Overall, though, preference shocks do not seem essential to understand the evolution of cross-sectional moments over the life-cycle and the time-series.

When we set $v_\varphi = 0$, instead, we reach the opposite conclusion. Figure 18 displays the best fit the model can achieve in absence of preference heterogeneity. The first major problem of this version of the model is its inability to generate a negative covariance between hours and wages and a positive covariance between consumption and wages. The former requires $\gamma > 1$, but for $\gamma > 1$ the latter covariance tends to be negative. In absence of preference heterogeneity, the model can generate a large variance of hours only when $\sigma$ is low. However, a high Frisch elasticity means that the variance of hours and the covariance between hours and wages are very sensitive to insurable shocks, and as a result the model overestimates the rise in both moments over this period.

7 Conclusions

Due to the work-in-progress nature of this project, rather than drawing conclusions we prefer to indicate directions that we intend to explore in the near future.

From an empirical perspective, we plan to use our set up to revisit the Abowd-Card criticism of the neoclassical labor supply model. Our framework gives a structural interpretation to the Abowd and Card statistical model, and allows us to re-examine their conclusion that the Frisch labor supply elasticity for men ought to be zero. We also plan
to show how the wage process can include a “predictable profile” which may be heterogeneous across agents, as long as the average growth rate of wages is the same across groups/islands.

Given the closed-form expressions for individual consumption and labor supply as a function of productivity and preference shocks, one can write down explicitly the likelihood function for individual histories. Then, using the approach of Blundell, Pistaferri and Preston (2005) a measure of total non-durable consumption can be imputed to PSID households drawing from information on the demand for food in the CEX. This augmented PSID panel can then be used for the likelihood estimation.

From a theoretical perspective, the current set up could be further extended in several directions. First, one can derive the same set of moments for non-separable preferences of the Cobb-Douglas class (as done in Heathcote, Storesletten and Violante, 2005b, with a simpler productivity process). Second, one can link the variances of insurable and uninsurable innovations to the aggregate state of the economy. Within this extended one can estimate the cyclicality of idiosyncratic risk and quantify its role for asset pricing (as in Constantinides and Duffie 1996, and Storesletten, Telmer and Yaron 2005), as well as for the welfare costs of business cycles. With respect to the latter issue, interestingly, fluctuations in the size of insurable and uninsurable individual risk will impact aggregate output through their effect of individual labor supply decisions, even without any shocks at the aggregate level. For example, times of large insurable uncertainty will be “good times” in the cycle. Third, it is possible to introduce taxes and transfer in a way that maintains tractability and allows for the explicit study of the impact of taxation on the cross-sectional moments of interest. Fourth, under some conditions, we can allow for a discrete participation decision. Finally, it would be desirable to have the flexibility to introduce in our framework a “spouse” with a separate labor supply decision from the “head”.
References


[34] Huggett, M., G. Ventura, and A. Yaron (2006b),"Sources of Lifetime Inequality", *Mimeo*, Georgetown University.


Figure 1: Life-cycle dimension – Macro facts
Figure 2: Life-cycle dimension – Macro facts
Figure 3: Life-cycle dimension – Micro facts
Figure 4: Time series dimension – Macro facts
Figure 5: Time series dimension – Macro facts
Figure 6: Time series dimension – Micro facts
Figure 7: Estimated variances of uninsurable and insurable innovations
Figure 8: Life-cycle dimension – Model’s fit
Figure 9: Life-cycle dimension – Model’s fit
Figure 10: Life-cycle dimension – Decomposition
Figure 11: Life-cycle dimension – Decomposition
Figure 12: Time series dimension – Model’s fit
Figure 13: Time series dimension – Model’s fit
Figure 14: Time series dimension – Model’s fit
Decomposition of the Variance of Wages

Decomposition of the Variance of Hours

Decomposition of the Variance of Earnings

Decomposition of the Variance of Consumption

Figure 15: Time series dimension – Decomposition
Figure 16: Time series dimension – Decomposition
Figure 17: Sensitivity analysis – Estimated cohort effects
Figure 18: Sensitivity analysis – Fit without preference shocks

59
Figure 19: Sensitivity analysis – Fit without preference heterogeneity