Technological Progress, Luck, and the Variability of Earnings

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Abstract

This paper develops a theoretical model to analyse how the arrival of a new technology embodied in capital affects the variability of earnings when workers’ adaptability to new technologies is subject to random events (luck) that can be history dependent. It is argued that technological change can leverage the importance of these stochastic factors and increase earnings instability in three ways. First, a rise in the speed of embodied technological progress raises the market premium to the lucky workers adaptable to the leading-edge technology. Second, the generality of the technology raises the ability of adaptable workers to transfer recently acquired knowledge to new machines and reduces the cost of retooling old machines, which increases the demand for adaptable workers. Third, the reduction in organizational knowledge about mature machines reduces the wage of those unlucky workers that cannot operate leading-edge machines.
1 Introduction

The technological improvements embodied in information and communication equipment developed over the last 30 years around the discovery of semiconductors have been truly impressive. Economists, scientists and organizational theorists have isolated three key features of this recent wave of technical change.

First, the acceleration in the speed of quality improvements in the equipment embodying new technology. Cummins and Violante (2002), building on Gordon (1990), document that the rate of embodied technical change averaged 4 percent per year in the past half-century, and since the mid-1970s, the pace of embodied technological improvement has accelerated from roughly 3.2 percent until 1975, to a rate of 4.9 percent after 1975. In the 1990s the growth has been spectacularly high, reaching an average annual rate of over 6 percent. The lion’s share of the acceleration is obviously attributable to computers, communication equipment other information processing goods: for example, in the period 1985-1996 the quality-adjusted price indexes for memory chips and microprocessors declined at an annual rate of 20%, and 35% respectively (Grimm, 1998), numbers which were just not imaginable thirty years ago.

Second, many authors have emphasized the “general purpose nature” of the new technology. Key electronic components are now incorporated into a large array of goods. A recent report of the Computer Science and Telecommunications Board of the National Academy of Sciences confirms the view of information technologies as GPT’s as it states that “...increased processing power can also often be used to [...] increase flexibility and generality, attributes that are key to much of the ongoing transformation of communication technology” (page 116). Similarly, a recent survey by the Bureau of Labour Statistics concludes that the impact of computers has been extensive because technologies, network systems, and software are similar across firms and industries, in contrast to technological innovations of the past which often affected specific occupations and industries, e.g. machine tool automation only involved production jobs in manufacturing (McConnell, 1996).

Prior to this paper, Greenwood and Yorukoglu (1997) used Gordon’s data to show that the growth rate of embodied technical change was 3% on average between 1954 and 1974 and 4% on average between 1974 and 1984. Hornstein and Krusell (1996) and Krusell, Ohanian, Ríos-Rull and Violante (2000) extended the series until 1992 and reached a similar conclusion.

Bresnahan and Trajtenberg (1995) coined the term “general purpose technologies” (GPT) to describe certain drastic innovations that have the potential for pervasive use and application in a wide range of sectors of the economy. Lipsey et al. (1998) cite as examples of such innovations, in ancient times, writing and printing, and in more recent times, the steam-engine, the electric dynamo and last, the microchip.
Third, scholars in organization and management science have noticed that the adoption of new information technologies often leads to a fall in labor productivity because it is associated to severe organizational transformations such as developing and customizing software, implementing new business processes, and changing work practices (Brynjolfsson and Hitt, 2000). Recently, Hugget and Ospina (2001) found robust evidence that the effect of a large equipment purchase is initially to reduce plant-level total factor productivity growth. In a nutshell, the process of organizational learning that takes naturally place in plants as they grow old can be slowed substantially when a new technology is introduced.\(^3\)

At the same time as this new technological breakthrough developed, important changes in the wage distribution took place in the US economy. Wage inequality has grown rapidly over this period, reaching arguably the highest peak in the postwar era. The empirical literature has documented that the fraction of the increase in inequality attributable to a rise in the return to permanent components of individual skills, such as educational attainment, age, and unobserved innate ability can explain between half and two thirds of the surge in inequality. The remaining part is related to the transitory components of earnings, i.e. it is due to an increase in the degree of earnings’ volatility and instability over the working life of observationally equivalent individuals (Katz and Autor, 2000). The quantitative importance of this transitory component has been documented extensively in a variety of ways. Using PSID data, Gottschalk and Moffitt (1994, 1997) decompose the increase in earnings inequality into a temporary and a permanent component and find that the rise in earnings instability due to transitory shocks is as large as the rise in the permanent component from 1970 to 1987. Gittleman and Joyce (1996) use matched cross-sections from the CPS to examine changes in earnings mobility from 1967 to 1991. They conclude, in agreement with Gottschalk and Moffitt, that short-term earnings mobility did not decline over the period. Blundell and Preston (1999) exploit income and consumption information from the CEX for the period 1980-1995 and identify only a minor upward trend in consumption inequality within educational and age groups, suggesting that the bulk of rising earnings inequality within-group is largely insurable, therefore fairly transitory in nature.\(^4\)

These sharp changes in the wage structure have sprung intense debates among economists.

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\(^3\)Aghion and Howitt (1998), Greenwood and Yorukoglu (1997) and Hornstein and Krusell (1996) provide theoretical frameworks where technological improvements can explicitly lead to a productivity slowdown through learning effects.

\(^4\)Similar results were found for Canada and the UK. Baker and Solon (1999) report that the rise in Canadian inequality has stemmed from upward trends in both the temporary and the permanent component, with the permanent component playing a somewhat larger role. Dickens (2000) studies the dynamic structure of male wages in the UK for 1975-1995 and concludes that the transitory component explains about half of the rise in inequality. The findings of Blundell and Preston (1999) for the U.K. are similar to those for the U.S..
The growing literature on the subject led to substantial progress in narrowing down the quest for robust explanations, in particular by emphasizing the primary role of technological progress (see Acemoglu, 2000 and Aghion 2001 for surveys). However, the existing theoretical literature has disproportionately emphasized the link between technological change and the increase in the return to the permanent component of skills (Acemoglu 1997, Heckman, Lochner and Taber 1998, Caselli 1999, Jovanovic 1998, Lloyd-Ellis 1999, Galor and Moav 2000), considering the rise in the transitory component somewhat of a puzzle.\(^5\) The objective of this paper is to develop a tractable dynamic general equilibrium model to analyze how the transitory component of earnings inequality (i.e. earnings variability) is affected by a new wave of embodied technological change characterized by all the three features described above. In the model, earnings are volatile along the life of ex-ante equal workers because the ability to move towards better job opportunities (technologies) is subject to stochastic factors that are history dependent: labour market history is scattered with stochastic events related to the luck of individuals, firms or industries. We argue that the rapid diffusion of a new technology leverages the importance of these stochastic factors, raising the premium to workers with no observable distinguishing characteristics other than their good fortune and increasing overall earnings instability.

The mechanism of the model can be explained easily. In the benchmark economy workers are ex-ante equal. Technological change takes place each period and is embodied in new machines. Wage inequality arises because some workers are fortunate enough to be adaptable to work with the most recent vintage of machines. Those who are adaptable two periods in a row earn an additional premium because they can employ skills on the new machines that were learned on last period’s new machines. The diffusion of a new technology raises the transitory component of earnings inequality through three separate channels. First, because of the technological acceleration, it allows lucky workers to work on more productive machines. Second, because of its general nature, it raises the skill transferability of those who are adaptable twice in a row. Moreover, the same general nature permits old machines to be retooled more easily to work with the new technology, thus amplifying the demand for adaptable workers. Third, because of lower organizational learning, the reduction in knowledge cumulated about mature machines further reduces the wage of those unlucky workers that cannot operate leading-edge machines.

The rest of the paper is organized as follows. Section 2 describes the economic environment

\(^5\)An exception is Violante (2002) which analyzes the link between the speed of technical change and the degree of skill transferability and looks at the quantitative implications of a technological acceleration for the rise in wage inequality in the US economy. Our paper is complementary, as it develops a more general theoretical framework, which is analytically tractable and derives a number of qualitative results. See also Gould, Moav and Weinberg (2001) for a model which explicitly distinguishes between the permanent and the transitory component of earnings inequality.
and the stationary competitive equilibrium. Section 3 analyzes the baseline model in which all workers are ex ante equal, and capital is putty-clay. Section 4 extends the model to allow for flexible capital adjustment. Section 5 concludes the paper.

2 The Economic Environment

2.1 Technology

Time is discrete, and indexed by $t$. Firms produce a good that can be used for consumption or for investment (machines). There is only one kind of consumption good, but each period an exogenous innovation occurs that allows a new improved vintage of machine to be produced. At any date only machines of the most recent vintage ("leading-edge" machines) can be produced.\(^6\) Thus output at date $t$ is:

$$y_t = c_t + k_t,$$

where $c_t$ denotes the amount produced of the consumption good and $k_t$ denotes the number of machines produced of vintage $t$.

Output is produced using labour and machines. Each machine lasts for two periods, with no depreciation taking place after the first period.\(^7\) Thus at any date $t$ there are two producing sectors; firms in sector 0 use leading-edge machines, while firms in sector 1 use "mature" machines (machines of vintage $t-1$).\(^8\) The production function in each sector is Cobb-Douglas with constant returns to scale. The (labour-augmenting) productivity parameter in sector 0 at date $t$ is $A_t = (1 + \gamma)^t$, where $\gamma > 0$ is the constant, exogenous rate of labour-augmenting technological progress. The productivity parameter in sector 1 at date $t$ is $(1 + \eta) A_{t-1}$, where $\eta > 0$ is the constant, exogenous rate of learning by doing.

It is prohibitively costly to retool mature machines so as to transform them into leading-edge machines.\(^9\) Thus the number of mature machines used in sector 1 at $t$ is $k_{t-1}$, and the aggregate production function is:

$$y_t = (A_t x_{0t})^{1-\alpha} k_t^\alpha + ((1 + \eta) A_{t-1} x_{1t})^{1-\alpha} k_{t-1}^\alpha, \quad t = 1, 2, \ldots$$

where $x_{it}$ is the labour input used in sector $i$, $i = 0, 1$, and $0 < \alpha < 1$.

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\(^6\)This assumption is relaxed in Section 4 below.
\(^7\)Our results generalize to the case of partial physical depreciation after one period.
\(^8\)This particular assumption is also made by Galor and Tsiddon (1997), although in the context of a model of wage inequality based upon differences in innate ability and in parental human capital.
\(^9\)This assumption is also relaxed in Section 4 below.
2.2 Preferences and Endowments

The economy is populated by a continuum of ex-ante identical, infinitely-lived individuals of mass 1. Perfect risk-sharing markets yield the same consumption profile for each individual, whose utility depends only on that profile, with unitary elasticity of intertemporal substitution:\(^10\)

\[
U = \sum_{t=1}^{\infty} \beta^{t-1} \ln c_t, \quad 0 < \beta < 1.
\]

Each individual is endowed with a unit of labour services at each date, and supplies it inelastically to one sector. Let \(n_{ij}\) denote the measure of individuals supplying labour to sector \(j\) this period, after having supplied sector \(i\) last period. The supply of labour to sector 1 is then:

\[
x_1 = n_{01} + n_{11}. \tag{2}
\]

An individual that worked in sector 0 last period acquired some knowledge that can be transferred to the leading edge this period; accordingly this individual’s unit of labour services can provide \(1 + \tau\) units of labour input to sector 0 this period, where \(\tau \in [0, \eta]\) is an exogenous “transferability” parameter. Accordingly, the supply of labour to sector 0 is:

\[
x_0 = (1 + \tau)n_{00} + n_{10}. \tag{3}
\]

Any individual can work in sector 1 where production takes place with mature machines. But not everyone can work in sector 0. Specifically, someone who worked in sector \(i\) last period has a probability \(\sigma_i\) of being able to work in sector 0 this period, \(i = 0, 1\). The probabilities \(\sigma_0\) and \(\sigma_1\) are exogenous constants reflecting the stochastic forces determining a person’s adaptability to the latest innovation. Assume \(\sigma_1 \leq \sigma_0\), so that experience on a more recent vintage improves the ability to work in a new technological environment, and hence increases the likelihood of being productive on the leading edge machines.

The law of large numbers implies that no more than a fraction \(\sigma_0\) (\(\sigma_1\)) of the labour force currently employed on leading-edge (mature) machines can be productively employed by firms operating leading-edge machines next period. More formally, the following aggregate adaptability constraints apply:

\[
\begin{align*}
n_{00} &\leq \sigma_0(n_{00} + n_{10}), \\
n_{10} &\leq \sigma_1(n_{01} + n_{11})
\end{align*}
\]

\(\text{(AC)}\)

\(^{10}\)Our results generalize to any constant elasticity of substitution.
2.3 Commentary

Our specification with respect to learning-by-doing and transferability reflects some implicit assumptions concerning technological knowledge. Thus the fact that everyone’s labour services are equally productive in sector 1, whether or not they are experienced with that vintage of machine, reflects the assumption that it is the owners of the machines, or firms, that are learning, rather than the workers, and what they are learning is “organizational knowledge” on how best to employ workers on their machines. This view of learning by doing accords, among others, with the study by Bahk and Gort (1993), who show that, after controlling for quality of capital and labour, the productivity of a plant still rises for several years after its birth.\textsuperscript{11}

When a particular kind of machine is new however, firms still don’t know how to exploit its full potential, and a worker who has experience with a similar machine may be able to operate the machine more efficiently than a worker without such experience. But the knowledge gained from experience on a very old vintage does not help much in this respect.\textsuperscript{12} Hence our specification that workers coming to sector 0 from sector 0 carry some extra skills, but not workers coming to sector 0 from sector 1. The requirement that $\tau < \eta$ reflects the assumption that prior experience on similar machines (of a prior vintage) is not as valuable as is long experience on identical machines (of the same vintage).

The transferability parameter reflects the “generality” of technological knowledge. An economy where $\tau = 0$ is one in which subsequent innovations are fundamentally different from each other, hence knowledge is completely specific to a particular innovation. The opposite extreme where $\tau = \eta$ corresponds to an economy in which technological knowledge is completely general; knowledge acquired through experience with the previous vintage is fully transferable on the new generation of machines.\textsuperscript{13}

As explained above, our assumption of history-dependent and stochastic individual adaptability rates differs sharply with the standard assumption in the literature of a permanent skill component which fully determines each worker’s ability to work with a new vintage (as in Caselli, Galor and Moav, Lloyd-Ellis, etc.). Our model is general enough, however, to encompass the innate ability case, a case with random iid adaptability shocks and every degree of persistence in between. When $\sigma_0 = 1$ and $\sigma_1 = 0$, the initial assignment of workers to vin-

\textsuperscript{11}In Aghion, Howitt, Violante (2001) we analyze the more traditional case where learning takes place at the individual level.

\textsuperscript{12}The last two sentences apply to the vintage model of learning by doing presented by Jovanovic and Nyarko (1996).

\textsuperscript{13}In the model of Chari and Hopenhayn (1991), knowledge is also vintage-specific, but it is embodied in workers ($\eta$ is attached to the worker, not the firm) and it is fully specific, so non-transferable across vintages ($\tau = 0$). Moreover, their focus is on technology diffusion, rather than on inequality.
tages repeats forever at the individual level, as an innate ability model would predict. When \( \sigma_0 = \sigma_1 = \sigma \) the adaptability rate at the individual level is not history dependent, but purely iid over time. The intermediate case of \( 1 > \sigma_0 > \sigma_1 > 0 \) generates history dependence in the likelihood that any worker can operate a leading-edge machine. This general approach allows us to analyze whether the diffusion of a technology generates more or less inequality according to the degree of persistence of individual adaptability parameters.

Finally, a remark on the relationship between \( \sigma \) and \( \tau \). The parameters \( \sigma_0 \) and \( \sigma_1 \) are indexes of the extensive adaptability margin of an economy, as they determine how many “bodies” the labour market is able to match productively with the latest vintage. The parameter \( \tau \) is an index of the intensive margin, as it determines how much skill each adaptable worker can productively transfer to the latest vintage. Interestingly, it will turn out that \( \sigma \) and \( \tau \) have contrasting effects on inequality.

2.4 Decisions of Workers and Firms and Stationary Equilibrium

We focus attention on a steady-state general competitive equilibrium with complete insurance markets, in which the labour flows \( n_{ij} \) are all constant from one period to the next; output, consumption and machinery all grow at the rate of technological progress \( \gamma \); the real rate of interest \( r \) is constant; the real wage rate \( w_{it} \) in each sector \( i \) grows at the rate \( \gamma \); and future values of all these variables are perfectly foreseen. The real rate of interest is governed by the steady-state Euler equation:

\[
\frac{1}{1+r} = \beta \frac{c_t}{c_{t+1}} = \beta \frac{1}{1+\gamma}.
\]

Let \( \omega_i = w_{it}/A_t \) denote the steady-state productivity-adjusted wage in each sector \( i \). Since firms are perfectly competitive, their aggregate input-output sequence \( \{k_t, x_{0t}, x_{1t}, y_t\}_{t=1}^{\infty} \) must maximize the present value of aggregate profits:

\[
\sum_{t=1}^{\infty} \left( \frac{1}{1+r} \right)^{t-1} (y_t - k_t - A_t \omega_0 x_{0t} - A_t \omega_1 x_{1t})
\]

subject to (1), with \( k_0 \) given. The first-order conditions for profit-maximization imply that the
relative wage in sector 0 is given by the ratio of marginal products:\(^{14}\)
\[
\frac{\omega_0}{\omega_1} = \frac{1 + \gamma}{(1 + \eta)^{1-\alpha}} \left( \frac{x_0}{x_1} \right)^{-\alpha}.
\]  

(5)

Since an individual who has worked in the leading-edge sector 0 for two periods in a row provides \((1 + \tau)\) units of labour on his current job, that individual’s productivity-adjusted wage is:
\[
\omega_{00} = (1 + \tau) \omega_0
\]  

(6)

A worker’s only labour-supply decision is whether to work in sector 0 or sector 1. This decision is made on the basis of wealth-maximization. Wealth equals the expected present value of lifetime wages, which is \(V_{1t} = A_t v_1\) for an individual working in sector 1 this period and \(V_{i0,t} = A_t v_{i0}\) for an individual working in sector 0 this period who worked in sector \(i\) last period. Productivity-adjusted wealth \(v\) obeys the Bellman equations:
\[
\begin{align*}
\nu &= \nu_0 + \beta \{ v_1 + \sigma_0 (\nu_0 - v_1)^+ \} \\
\nu_0 &= \nu_0 + \beta \{ v_1 + \sigma_0 (\nu_0 - v_1)^+ \} \\
\nu_1 &= \nu_1 + \beta \{ v_1 + \sigma_1 (\nu_1 - v_1)^+ \}
\end{align*}
\]

where the notation \(x^+\) indicates the maximum of \(x\) and 0, and where we have made use of the Euler equation (4) determining the rate of interest. So, for example, a worker coming from sector 0 to sector 0 earns a productivity-adjusted wage \(\nu_{00}\) and has a guaranteed continuation value of \((1 + \gamma) v_1\), since working in sector 1 is always an option. With probability \(\sigma_0\) the worker will have the option of continuing in sector 0 next period, which will be exercised if \(\nu_{00} > v_1\).

Wealth-maximization by individual workers subject to the adaptability constraints (AC) implies the complementary inequalities:
\[
\begin{align*}
0 &\leq n_{00} & \text{with } = & \text{ if } v_1 > v_{00} \\
n_{00} &\leq \sigma_0 (n_{00} + n_{10}) & \text{with } = & \text{ if } v_1 < v_{00} \\
0 &\leq n_{10} & \text{with } = & \text{ if } v_1 > v_{10} \\
n_{10} &\leq \sigma_1 (n_{01} + n_{11}) & \text{with } = & \text{ if } v_1 < v_{10}
\end{align*}
\]

\(^{14}\)The first-order conditions with respect to \(x_{0t}\) and \(x_{1t}\) are:
\[
\omega_0 = (1 - \alpha) (A_t x_{0t}/k_t)^{-\alpha}
\]

and
\[
\omega_1 = (1 - \alpha) \frac{(1 + \eta)^{1-\alpha}}{1 + \gamma} (A_{t-1} x_{1t}/k_{t-1})^{-\alpha}.
\]

Equation (5) follows from these and the stationarity condition that \(x_{0t}, x_{1t}\) and \(k_t/A_t\) be constant.
So, for example, if $v_1 < v_{00}$ then everyone who worked in sector 0 last period and can do so this period will choose to do so.

It is useful to highlight the economic factors behind the workers’ mobility decisions. The choice of moving onto a new technology involves a trade-off: in terms of current pay-off, a new technology guarantees higher efficiency of capital (by a factor $\gamma$), but lower knowledge and experience on the new technology. As for the continuation value, being on a new technology always gives the worker a higher adaptability rate, and the option of larger future skill transferability.

Labour-market clearing requires full employment:

$$n_{00} + n_{10} + n_{01} + n_{11} = 1. \quad (7)$$

Furthermore, in a steady state the flows of labour into and out of sector 1 must be equal:

$$n_{01} = n_{10}. \quad (8)$$

### 3 Equilibrium Earnings Variability

We measure earnings variability by the ratio between the highest and the lowest wage that a worker can receive over her lifetime in this economy:

$$R_\omega = \frac{\max \{\omega_{00}, \omega_0, \omega_1\}}{\min \{\omega_{00}, \omega_0, \omega_1\}}$$

This wage ratio index $R_\omega$ has the advantage of being simple to characterize and of behaving in the vast majority of the cases similarly to other more common measures of wage variability. In Section 3.3 we report a comparison between the $R_\omega$ index, and two alternative measures of variability of earnings, the variance of log-wages ($V_\omega$), and the 90/10 percentile wage ratio ($D_\omega$) and we show that all our conclusions are robust to these alternative measures. It goes without saying that, since workers are ex-ante identical and the heterogeneity due to limited adaptability is stochastic and iid across individuals, each of these statistics also measures the degree of cross-sectional wage inequality in the economy. This cross-sectional wage inequality is entirely transitory in the sense we have explained above.

The equilibrium condition (6) determines the relative wage $\omega_{00}/\omega_0$ of the two kinds of worker in sector 0. A complete description of all relative wages can thus be obtained by determining the relative wage $\omega_0/\omega_1$ between the two sectors. Equation (5) describes a “relative demand curve” relating $\omega_0/\omega_1$ to the relative quantity $x_0/x_1$. We determine $\omega_0/\omega_1$ by putting (5) together
with a “relative supply curve,” which we now proceed to construct on the basis of the previous Section’s analysis.

First, note that individuals who worked in sector 1 last period will be indifferent between the two sectors this period if the relative wage \( \omega_0/\omega_1 \) equals:

\[
\Omega = \frac{1}{1 + \beta \sigma_0 \tau},
\]

for then \( v_1 = v_{10} \). In this case, workers coming from sector 0 last period will strictly prefer to stay in sector 0, since \( v_{00} = v_{10} + \tau \omega_0 > v_{10} \), so their adaptability constraint will be binding. By the same reasoning, when \( \omega_0/\omega_1 > \Omega \), every worker will strictly prefer to work in sector 0, and both adaptability constraints will be binding. In this case, the relative supply \( x_0/x_1 \) equals:\footnote{With both adaptability constraints binding, we have \( n_{10} = \sigma_1 (n_{01} + n_{11}) \) and \( n_{00} = \sigma_0 (n_{00} + n_{10}) \). These two equations and the two labour market equilibrium conditions (7) and (8) can be solved for the four labor flows \( n_{ij} \)’s. Substituting the solutions into relationships (2) and (3) yields (9).}

\[
\chi = \frac{\sigma_1}{1 - \sigma_0} (1 + \sigma_0 \tau). \tag{9}
\]

Thus the relative supply curve has the reverse-L shape depicted in Figure 1. When the relative wage falls below \( \Omega \), workers coming from sector 1 strictly prefer to stay there, so that the steady-state flow into sector 0 is 0, which implies \( x_0 = 0 \).\footnote{That is, together with (7) and (8), \( n_{10} = 0 \) implies \( n_{00} = 0 \) and therefore, by (3), \( x_0 = 0 \).} When the relative wages equals \( \Omega \) then wealth-maximization allows the fraction of workers from sector 1 that flow into sector 0 to be anything between 0 and \( \sigma_1 \), which is consistent with any relative supply between 0 and \( \chi \).

**Figure 1 here**

Define \( \Phi \) as the relative wage along the relative demand curve when the relative supply is \( \chi \):

\[
\Phi = \frac{(1 + \gamma)}{(1 + \eta)^{1 - \alpha}} \left[ \frac{(1 - \sigma_0)}{\sigma_1 (1 + \sigma_0 \tau)} \right]^\alpha
\]

If \( \Phi \geq \Omega \) then both adaptability constraints bind and \( \omega_0/\omega_1 = \Phi \), as illustrated by demand curves \( D_a \) and \( D_b \) in Figure 1. Otherwise \( \omega_0/\omega_1 = \Omega \). Thus:

\[
\omega_0/\omega_1 = \max \{\Omega, \Phi\}. \tag{10}
\]

The maximal wage is always \( \omega_{00} \), because otherwise wages in sector 1 would dominate those in sector 0 and the relative supply \( x_0/x_1 \) would be zero.\footnote{More formally, \( \omega_{00}/\omega_0 = 1 + \tau > 1 \) and \( \omega_{00}/\omega_1 = (\omega_{00}/\omega_0) (\omega_0/\omega_1) = (1 + \tau) (\omega_0/\omega_1) \geq (1 + \tau) \Omega = (1 + \tau) / (1 + \beta \sigma_0 \tau) > 1 \).} When \( \Phi \geq 1 \), then the minimal wage
is $\omega_1$ and $R_\omega = \omega_{00}/\omega_1 = (\omega_{00}/\omega_0)(\omega_0/\omega_1) = (1 + \tau)\Phi$. When $\Phi < 1$, then, since $\Omega < 1$, the minimal wage is always $\omega_0$ and $R_\omega = \omega_{00}/\omega_0 = (1 + \tau)$. Putting these results together and using straightforward differentiation yields:

**Proposition 1** In the basic model,

1. The wage ratio index ($R_\omega$) in steady-state equilibrium is given by:

$$R_\omega = (1 + \tau) \max \{1, \Phi\}.$$ 

where $\Phi$ is defined above.

2. Moreover:

$$\frac{\partial R_\omega}{\partial \tau} > 0, \quad \frac{\partial R_\omega}{\partial \gamma} \geq 0, \quad \frac{\partial^2 R_\omega}{\partial \gamma \partial \tau} \geq 0, \quad \frac{\partial R_\omega}{\partial \sigma_i} \leq 0, \quad \frac{\partial R_\omega}{\partial \eta} \leq 0.$$ 

In particular, if $\gamma$ is small enough relative to $\eta$ then $R_\omega = 1 + \tau$. In this region of the parameter space, any increase in $\gamma$ will affect the wage differential between workers on different vintages, but it will leave the maximal wage spread unaffected, which in turn occurs within sector 0; thus $R_\omega$ will be insensitive to the speed of technological progress. However, when $\gamma$ is large then $R_\omega = (1 + \tau)\Phi$. In this region any increase in $\gamma$ will raise the maximal wage spread, which now occurs between sectors, so $R_\omega$ will be sensitive to changes in $\gamma$. An increase in $\tau$ will always raise wage instability $R_\omega$ because it always raises the premium earned by the highest paid workers, namely those transferring skills from the previous leading edge to the current leading edge. Figure 2 provides a graphical representation of $R_\omega$ as a function of the pair $(\gamma, \tau)$, and clearly shows the two regions.

**Figure 2 here**

### 3.1 The Nature of Technical Change and the Rise in Earnings Variability

Analyzed through Proposition 1, the arrival of a new technology can be seen as having several distinct effects on wage variability through the changes it induces on the parameters $\gamma, \tau, \sigma_i, \eta$. We have discussed in the Introduction the three main features of the breakthrough of a new technology.

First, such a breakthrough accelerates the rate of technological change *embodied* in equipment investment (higher $\gamma$). In the benchmark model earnings variability increases with the rate of embodied technical progress $\gamma$: the higher $\gamma$, the bigger the wedge between the wage a worker will receive in the event that she can adapt to new innovations and the wage paid in
the unlucky event she cannot. Note that the positive effect of faster technological progress on wage instability is leveraged by the level of transferability \( \tau \).

Second, the new technological paradigm leads to a process whereby productive resources within plants are diverted towards costly organizational transformation which (at least temporarily) slow down the natural learning curve of those plants (lower \( \eta \)). In the model, wage volatility decreases with the rate of learning-by-doing \( \eta \): the lower \( \eta \) the less knowledge is cumulated about mature machines. This force decreases the wage paid to workers when they cannot operate leading-edge machines, which is often the lowest paid state in the economy.

Third, because of its “general nature”, a new technological platform makes successive vintages of capital more similar to each other; thus it increases skill transferability towards new technologies (higher \( \tau \)) and the adaptability of workers to new vintages (higher \( \sigma_i \)).\(^{18}\) According to Proposition 1, the first of these economic forces works towards increasing wage variability along individual working lives, even though the expression for \( R_\omega \) in Proposition 1 uncovers two counteracting effects when both adaptability constraints bind and \( \omega_0 / \omega_1 = \Phi \). The multiplicative term \((1 + \tau)\) reveals a direct effect for given labour supply: the higher \( \tau \) the greater the comparative advantage of moving from leading edge to leading edge. The term in the denominator \((1 + \sigma_0 \tau)\) captures an indirect general equilibrium effect: the higher \( \tau \), the bigger the supply of labour in the leading edge sector (i.e. the larger the ratio \( x_0 / x_1 \)) which in turn tends to lower the relative wage of these workers and therefore the ratio \( R_\omega \). By simple differentiation, it is easy to see that the direct effect of \( \tau \) always dominates. Conversely, earnings instability decreases with the adaptability rates \( \sigma_i \): the higher \( \sigma_i \), the bigger the potential supply of workers that can adapt to new innovations and therefore the smaller the premium earned by workers in the lucky event they are adaptable.

The overall effect cannot be determined \textit{a priori}, yet the only effect leading to lower inequality (higher \( \sigma_i \)), disappears in the parameter region where maximal inequality occurs \textit{within} rather than \textit{between} sectors. Specifically, it follows from Proposition 1 that when \( \Phi < 1 \), that is when:

\[
\left( \frac{\sigma_1 (1 + \tau \sigma_0)}{1 - \sigma_0} \right)^\alpha \geq \frac{1 + \gamma}{(1 + \eta)^{1-\alpha}}, \tag{11}
\]

then an increase in either of the \( \sigma_i \)'s has no effect on \( R_\omega \): the maximal wage spread then occurs \textit{within} the leading-edge sector 0, and therefore it remains unaffected by the mobility of workers between sectors.

In fact, equation (11) determines an upper bound \( \bar{\sigma} < 1 \) such that whenever \( \sigma_0 \) is bigger than \( \bar{\sigma} \), then increased mobility has no effect on \( R_\omega \). Since there is no such limit on the range of

\(^{18}\)A fourth effect, working through capital retooling, is analyzed in Section 4.3 below.
parameter values over which an increase in transferability (τ) has a positive effect on inequality, an overall negative effect of technological change on earnings variability can occur only if mobility is initially sufficiently small that it satisfies (11). Moreover, the larger τ the smaller \(\bar{\sigma}\), and therefore the less likely it is that the overall effect from the diffusion of a new technology on wage variability will be negative.

### 3.2 Individual Persistence in Adaptability and Wage Variability

One interesting question we can ask our model is: what is the relationship between persistence in adaptability at the individual level and earnings instability? In other words, how does individual persistence in adaptability affect the magnitude of the rise in the degree of earnings volatility, when the economy finds itself in the sensitive region of the parameter space?

Consider the two parameters \(\sigma_0\) and \(\sigma_1\). As argued above, \(\sigma_0 = 1\) and \(\sigma_1 = 0\) correspond to an innate ability model where initial conditions matter forever. On the contrary, when \(\sigma_0 = \sigma_1\) history dependence in mobility options disappears. To analyze the pure effect of persistence, we need to perform comparative statics on \(\sigma_0\) and \(\sigma_1\) assuming that the aggregate number of adaptable workers remains unchanged. The latter is simply equal to \(n_{10} + n_{00} = \frac{\sigma_1}{1 - \sigma_0 + \sigma_1} N\), which implies that the ratio \(\frac{\sigma_1}{1 - \sigma_0}\) must be constant. When we decrease persistence, by reducing \(\sigma_0\) keeping the ratio \(\frac{\sigma_1}{1 - \sigma_0}\) fixed, the wage spread measured by \(R_\omega\) unambiguously rises. It is also easy to see that lower persistence accelerates the rise in the maximum wage differential due to an increase in \(\gamma\) or/and in \(\tau\). This result is explained by the fact that as \(\sigma_0\) gets closer to \(\sigma_1\), the fraction of “low transferability” workers among movers rises, hence less skills are being transferred on to new technologies and the general equilibrium effect intrinsic in the denominator of \(\Phi\) is downplayed, which clearly rises the wage spread.

This is an interesting result because, as mentioned in the introduction, the bulk of the literature based on increasing returns to innate ability implicitly assumes \(\sigma_0 = 1\) and \(\sigma_1 = 0\). We show here that these models ignore an important mechanism associated with the randomness and history dependence in individual adaptability, and consequently they tend to underestimate the effect of technological changes on within-group inequality.

### 3.3 Alternative Measures of Wage Variability

It is important to verify that the results in Proposition 1 do not hinge upon our particular measure of wage variability, namely the wage ratio index \(R_\omega\), but that these results hold true for alternative (and more general) measures. We thus also consider the variance of log-wages.

\(^{19}\)However, notice that a rise in \(\gamma\) would increase the upper bound \(\bar{\sigma}\).
(\(V_\omega\)) and the 90 – 10 log wage differential (\(D_\omega\)). Since it is prohibitive to obtain simple closed forms for \(V_\omega\) and \(D_\omega\) in order to do analytic comparative statics, we have performed a number of simulations on the model to check the robustness of our conclusions.

The key results of these simulations are plotted in panels (1)-(6) of Figure 3. For these simulations, we have chosen parameter values that we regard as reasonable.\(^{20}\) Since we assumed that machines depreciate in two periods, a consistent choice of the length of the period would be five years. The capital share parameter \(\alpha\) is set to .3, and the discount factor \(\beta\) to .98 on an annual basis, implying an annual rate of return on capital of 5% when \(\gamma\) is 3% per year. In the benchmark case the learning rate \(\eta\) is set to .22, which corresponds to a yearly measure of returns to experience of 4%, a number within the range estimated in the literature. For the transferable knowledge \(\tau\), we take a baseline value of 10% (roughly half of the learning rate \(\eta\)), and for the adaptability rates, we assume \(\sigma_0 = \sigma_1 = .5\). We shall explore the joint behavior of \(R_\omega\), \(V_\omega\), and \(D_\omega\) for values of \(\gamma\) ranging from 0 to 7% per year, for values of \(\eta\) ranging from .1 to .3, for values of \(\tau\) in the interval between 0 and .22, and for values of \(\sigma\) between .3 and .7.

In the panels of Figure 3, we have changed the parameters one at the time with respect to the benchmark.

First, notice that \(V_\omega\) increases with \(\gamma\) always at a faster rate than \(R_\omega\) and \(D_\omega\). Second, as expected, \(V_\omega\) does not have a flat region with respect to \(\gamma\), however the simulations of Figure 3 show that in that same insensitive region of low values of \(\gamma\), \(V_\omega\) tends to grow more slowly than elsewhere.\(^{21}\) Third, only in the case of very large aggregate adaptability is the behavior of \(V_\omega\) different from that displayed by the other two measures, as the latter then become unaffected by faster technical change, while \(V_\omega\) keeps rising with \(\gamma\). Fourth, as can be seen in Figure 3, in all our numerical simulations the behavior of \(R_\omega\) and \(D_\omega\) coincide almost perfectly. Based upon this observed similarity, focusing on \(R_\omega\) does not seem to involve any major loss of generality or insight.

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\(^{20}\) This is not meant to be a calibration, but simply a numerical simulation to check the robustness of our conclusions in Proposition 1.

\(^{21}\) It should be said that for some extreme parametrization it is possible to generate, within this region of low \(\gamma\), small declines in \(V_\omega\). The reason is that, as soon as the economy switches from \(v_{10} < v_1\) to \(v_{10} > v_1\), the wage \(\omega_1\) which is the intermediate wage, starts falling and gets increasingly closer to \(\omega_0\), thereby reducing wage heterogeneity in the economy. This effect is not captured by the wage ratio index \(R_\omega\), since the intermediate wage is irrelevant for such measure.
4 The Economy with Flexible Capital Adjustment

In this section, we relax the putty-clay assumption of the baseline model. We do it in two steps. First, we allow firms to retool their old capital and convert it into capital embodying the leading-edge technology. Firms might have the incentive to do so when adaptable labor is relatively abundant. Second, we allow firms to revive the old capital, and invest in old technologies, which might happen if adaptable labour is particularly scarce. As one can expect, retooling and reviving have opposite effects on the relative demand for adaptable labor, hence on the maximum wage spread in the economy. In the last part of the section we argue that the diffusion of a new technology should make retooling (reviving) more (less) likely to happen and accelerate the rise in wage instability.

4.1 Retooling of Old Capital

Suppose that after a machine has been used for one period a firm may convert it into \((1 - \delta)\) leading-edge machines, where \(\delta \in [0, 1]\) is the exogenous cost of retooling. Although a retooled machine at \(t\) embodies the leading-edge productivity parameter \(A_t\), it is unlike a newly-produced leading-edge machines in that it will fully depreciate at the end of the period. Let \(k_{it}\) be the number of machines used in sector \(i\) at \(t\) and \(d_t\) be the number of mature machines that are retooled (into \((1 - \delta) d_t\) leading-edge machines) at \(t\). Then \(k_{1t}\) equals the number of newly-produced leading-edge machines at \(t - 1\) minus the number of mature machines retooled at \(t\):

\[
k_{1t} = k_{0t-1} - (1 - \delta) d_{t-1} - d_t, \quad t = 1, 2, ....
\]  

and the aggregate production function is:

\[
y_t = (A_t x_{0t})^{1-\alpha} k_{0t}^{\alpha} + ((1 + \eta) A_{t-1} x_{1t})^{1-\alpha} k_{1t}^{\alpha}, \quad t = 1, 2, ....
\]  

In equilibrium the sequence \(\{k_{0t}, k_{1t}, d_t, x_{0t}, x_{1t}, y_t\}_{t=1}^\infty\) must maximize the present value of aggregate profits:

\[
\sum_{t=1}^\infty \left( \frac{1}{1+r} \right)^{t-1} (y_t - k_{0t} - (1 - \delta) d_t - A_t \omega_0 x_{0t} - A_t \omega_1 x_{1t})
\]

subject to (12), (13) and a non-negativity constraint on each \(d_t\).

The introduction of retooling does not change the relative supply curve of Figure 1, but it does change the relative demand curve. Specifically, there is a critical relative wage:

\[
\Phi^{ret} = \frac{1 + \gamma}{(1 + \eta)^{1-\alpha} (1 - \delta)^{1-\alpha}}
\]
such that if $\omega_0/\omega_1 \geq \Phi^{ret}$ then firms will choose not to retool any mature machines and the relative demand curve will continue to be defined by equation (5), but once $\omega_0/\omega_1$ falls to the level $\Phi^{ret}$, retooling will occur and increase in the relative input of leading-edge labour will induce a proportional increase in the steady-state supply of leading-edge machines, with no change in relative wages.\textsuperscript{22} That is, the relative-demand curve will now be:

$$\frac{\omega_0}{\omega_1} = \max \left\{ \frac{1 + \gamma}{(1 + \eta)^{1-\alpha}} \left( \frac{x_0}{x_1} \right)^{-\alpha}, \Phi^{ret} \right\}$$

as illustrated in Figure 4.

\textbf{Figure 4 here}

If, as in the case depicted in Figure 4, $\Phi^{ret} > \max \{\Phi, \Omega\}$, then in equilibrium retooling will take place, both adaptability constraints will be binding and $\omega_0/\omega_1 = \Phi^{ret}$. Otherwise, the relative wage $\omega_0/\omega_1$ will equal $\max \{\Phi, \Omega\}$, as before. Thus:

$$\omega_0/\omega_1 = \max \{\Omega, \Phi, \Phi^{ret}\}, \quad (15)$$

and the same logic that led from (10) to Proposition 1 now leads from (15) to:

\textbf{Proposition 2} When retooling is an option:
1. The wage ratio index $(R_\omega)$ in steady-state equilibrium is given by:

$$R_\omega = (1 + \tau) \max \{1, \Phi, \Phi^{ret}\}.$$  

where $\Phi$ and $\Phi^{ret}$ are defined above.

2. Moreover:

$$\frac{\partial R_\omega}{\partial \tau} > 0, \quad \frac{\partial R_\omega}{\partial \gamma} \geq 0, \quad \frac{\partial^2 R_\omega}{\partial \gamma \partial \tau} \geq 0, \quad \frac{\partial R_\omega}{\partial \sigma_i} \leq 0, \quad \frac{\partial R_\omega}{\partial \eta} \leq 0,$$

$$\frac{\partial R_\omega}{\partial \delta} \leq 0, \quad \frac{\partial^2 R_\omega}{\partial \tau \partial \delta} \leq 0, \quad \frac{\partial^2 R_\omega}{\partial \gamma \partial \delta} \leq 0.$$

\textsuperscript{22}The first-order conditions for profit-maximization with respect to $x_{0t}$ and $x_{1t}$ imply:

$$\frac{\omega_0}{\omega_1} = \frac{1 + \gamma}{(1 + \eta)^{1-\alpha}} \left( \frac{x_0/k_0}{x_1/k_1} \right)^{-\alpha},$$

where $k_0 = k_{0t}/A_t$ and $k_1 = k_{1t}/A_{t-1}$, both constant in a steady-state. From (12) and the non-negativity constraint on $d_t$, $k_0 \geq k_1$, with equality if no retooling takes place. Thus:

$$\frac{\omega_0}{\omega_1} \geq \frac{1 + \gamma}{(1 + \eta)^{1-\alpha}} \left( \frac{x_0}{x_1} \right)^{-\alpha}, \text{ with equality if no retooling takes place.}$$

The first-order (Kuhn-Tucker) conditions with respect to $d_t$ and $k_i$ can be solved to yield:

$$\frac{\omega_0}{\omega_1} \geq \Phi^{ret}, \text{ with equality if retooling takes place.}$$
Thus transferability, speed, adaptability and learning by doing have the same qualitative effects as before on the degree of wage variability. The cost of retooling now has a negative effect on the wage spread. This is because a fall in $\delta$ encourages firms to employ more capital with adaptable workers who are already earning the maximum wage, and correspondingly less capital to those (in sector 1) already earning the minimum wage. Thus, as workers move between technologies, they will be subject to larger wage fluctuations.

Furthermore, the effects of retooling reinforce the effects of transferability and the speed of technological change. Consider, for example, an increase in $\tau$, in the case where retooling is taking place and both adaptability constraints are binding (that is, $\Phi^\text{ret} = \max\{1, \Phi, \Phi^\text{ret}\}$). Then the general equilibrium effect that we saw operating in the benchmark model (see item 3 in the discussion following Proposition 1) which tended to moderate the increase in transitory inequality will no longer operate; the increase in relative labour input to sector 0 workers will lead not to a dampening of relative wages in sector 0 but to a proportional increase in the number of machines employed in sector 0.

4.2 Reviving of Old Technology

In this section we relax the assumption that only leading-edge machines can be produced. Intuitively, this should reduce the relative demand for adaptable workers and therefore their relative wage in steady-state equilibrium. In other words, allowing firms to revive old technologies has the opposite effect from that of allowing old machines to be retooled using new technologies, namely it decreases the wage spread in the economy. Allowing for capital revival also introduces the interesting possibility that three instead of two subsequent vintages be operated at the same time.

Let $k_t$ denote the production of new machines (leading-edge plus mature) at $t$. Let $d_t$ be the number of mature machines retooled and $e_t$ be the number of mature machines produced at $t$. There is now a second sector, namely sector 2, in which very mature machines (two-period old vintage) are used. Anyone can work in this subsector, just as anyone can work in sector 1 where mature machines are used. Thus workers in each of these sectors must receive the same wage $\omega_1$ in a competitive equilibrium. Let $k_{0t}, k_{1t}, k_{2t}$ and $x_{0t}, x_{1t}, x_{2t}$ denote the inputs of machines and labour to the three sectors. Then:

\begin{equation}
  k_{0t} = k_t + (1 - \delta) d_t - e_t, \quad t = 1, 2, ...
\end{equation}

\begin{equation}
  k_{1t} = k_{t-1} - e_{t-1} + e_t - d_t, \quad t = 1, 2, ...
\end{equation}
\[ k_{2t} = e_{t-1}, \quad t = 1, 2, \ldots \] (18)

and the aggregate production function is:

\[ y_t = (A_t x_{0t})^{1-\alpha} k_{0t}^{\alpha} + ((1 + \eta) A_{t-1} x_{1t})^{1-\alpha} k_{1t}^{\alpha} + ((1 + \eta) A_{t-2} x_{2t})^{1-\alpha} k_{2t}^{\alpha}, \] (19)

The sequence \( \{x_{0t}, x_{1t}, x_{2t}, k_{0t}, k_{1t}, k_{2t}, k_t, d_t, e_t, y_t\}_{t=1}^{\infty} \) must maximize the present value of aggregate profits:

\[
\sum_{t=1}^{\infty} \left( \frac{1}{1+r} \right)^{t-1} (y_t - k_t - A_t \omega_0 x_{0t} - A_t \omega_1 (x_{1t} + x_{2t}))
\]

subject to (16) – (19) and non-negativity constraints on each \( d_t \) and \( e_t \).

As in the case where retooling but not reviving was an option, retooling will take place once the relative wage \( \omega_0/\omega_1 \) has fallen to \( \Phi^{ret} \), as defined by (14). But now reviving will take place once the relative wage has risen to:

\[
\Phi^{rev} = \frac{1 + \gamma}{(1 + \eta)^{1-\alpha}} \left( 1 + \frac{\beta}{1 + \gamma} \left( (1 + \gamma)^{\frac{\alpha-1}{\alpha}} - 1 \right) \right)^{\frac{\alpha}{\alpha-1}}.
\]

Notice that:

\[
\Phi^{rev} > \frac{1 + \gamma}{(1 + \eta)^{1-\alpha}} > \Phi^{ret}.
\]

When the relative wage lies between \( \Phi^{rev} \) and \( \Phi^{ret} \) the demand for labor continues to be given by equation (5). Thus the complete relative-demand schedule is as shown in Figure 5.

**Figure 5 here**

Assume that \( \Phi^{rev} > \Omega \). Then there will exist a stationary equilibrium with a positive relative supply \( x_0/x_1 \). Reasoning as before, we have:

**Proposition 3** When both retooling and reviving are options:

1. The wage ratio index \( R_\omega \) in steady-state equilibrium is given by:

\[
R_\omega = (1 + \tau) \max\{1, \Phi^{ret}, \min(\Phi, \Phi^{rev})\}.
\]

\[\text{23}\] The first-order conditions for profit-maximization with respect to \( k_t, k_{0t}, k_{1t}, k_{2t} \) and \( e_t \) can be solved to yield:

\[
\frac{\omega_0}{\omega_1} \leq \Phi^{rev}, \text{ with equality if reviving takes place.}\]
Moreover:

\[ \frac{\partial R_\omega}{\partial \tau} > 0, \quad \frac{\partial R_\omega}{\partial \gamma} \geq 0, \quad \frac{\partial^2 R_\omega}{\partial \gamma \partial \tau} \geq 0, \quad \frac{\partial R_\omega}{\partial \sigma_i} \leq 0, \quad \frac{\partial R_\omega}{\partial \eta} \leq 0. \]

Comparing the expressions for \( R_\omega \) in Propositions 2 and 3 shows that if the possibility of reviving has any effect on wage instability it is a negative one. This is because diverting resources from producing new leading-edge machines to producing new mature machines increases the potential demand for the non-adaptable workers who receive the minimum wage when both adaptability constraints bind. However, introducing reviving does not change the qualitative comparative statics results, thus the effects of a new technology diffusion highlighted for the benchmark model still hold true. In the next section we discuss some additional channels through which a new technology can impact earnings variability in the augmented model.

**4.3 The Impact of a New Technology on Capital Adjustment**

The augmented model allows us to highlight two additional effects of technological breakthroughs on earnings variability. First, an increase in the general nature of technology might decrease the cost of retooling machines (lower \( \delta \)). Second, if \( \gamma \) and \( \tau \) are sufficiently large during the acceleration phase of the diffusion of a new technology, then \( \Phi > \Phi^{rev} \), so that old vintage sectors will no longer operate. Thus, whilst an acceleration in the diffusion of a new technology encourages the retooling of old machines, it also discourages the reviving of old technologies, thereby inducing a magnified increase in earnings instability in comparison to the baseline case in which neither retooling nor reviving were not allowed.

Moreover, the augmented model provides us with two additional reasons for thinking that the only possible negative effect of a new technology on wage differentials, i.e. the one working through increased adaptability \( \sigma_i \), will be limited. Specifically, it follows from Proposition 2 and Proposition 3 that only when:

\[
\max \left\{ \left(1 - \frac{\alpha}{\delta} \right)^{\frac{\alpha}{\delta}}, \frac{1 + \gamma}{(1 + \eta)^{1-\alpha}} \right\} \geq \left( \frac{\sigma_1 (1 + \tau \sigma_0)}{1 - \sigma_0} \right)^{\alpha} \geq \left( 1 + \frac{\beta}{1 + \gamma} \left( (1 + \gamma)^{\frac{\alpha-1}{\alpha}} - 1 \right) \right)^{\frac{\alpha}{1-\alpha}} \]

then an increase in either of the \( \sigma_i \)'s lowers \( R_\omega \). Given any values of the other parameters, equation (11) places an upper bound \( \sigma < 1 \) and a lower bound \( \sigma > 0 \) on \( \sigma_0 \), beyond which adaptability has no further effect on the wage spread. First, as shown by (11) this factor is limited by the possibility that maximal inequality occurs within rather than between sectors. Moreover, (20) shows that it is also limited by the possibility of retooling and reviving. For very large values of \( \sigma_i \) adaptable labor is abundant, and firms retool old capital into new
technologies. An increase in $\sigma_i$ in this range generates a downward pressure on inequality which is fully offset by a rise in the capital stock employed with adaptable labor, through additional retooled machines. For small values of $\sigma_i$ firms produce old vintages of machines because the labour cost of working with leading-edge machines is too high. An increase in $\sigma_i$ in this range leads to an increase in the steady-state relative supply of adapted workers, but, as in the case where retooling takes place, this will result not in a change in the relative wage but in a proportional increase in the number of machines employed with adapted workers, this time because of reduced production of new mature (revived) machines.

5 Concluding Remarks

In this paper we have developed a simple dynamic general equilibrium model to analyze how the diffusion of a new technology –through its effects on the speed of embodied technological change, on the transferability of skills across sectors, and on the organizational costs related to its adoption– can affect the degree of earnings variability, i.e. the transitory component of wage inequality. Our model is based upon “luck": the adaptability of a worker to the new vintage of technology is determined by stochastic events along her working life which lead to divergent labor market histories for ex-ante identical workers. The breakthrough of a new technology amplifies the consequences of such random events and leads to an increase in the variability of individual earnings, a fact that has been documented extensively in the empirical literature (Gottshalk-Moffitt 1994, Blundell and Preston 1996, and Gittleman and Joyce 1999).

As explained in the Introduction, our model tries to explain the rise in the transitory component of inequality rather than the permanent component, which is instead the focus of much of the literature. This distinction is not just semantic, but it has important policy implications. To begin with, insofar as we are interested in reducing inequality, models in the first class call for interventions that allow the disadvantaged (or unlucky) workers to rebuild their skill level, especially the vintage of their knowledge. Models in the second class suggest that the intervention should be targeted much earlier in the life of an individual, possibly during childhood when the crucial components of cognitive ability are being formed. However, the difference in policy implications can be even starker: higher earnings instability is associated to larger income mobility, which can be perceived as a good thing from an incentive perspective. Conversely, higher long-run (permanent) inequality may be perceived as detrimental to economic stability and growth (Alesina and Rodrick, 1994).

The theoretical analysis developed in this paper has two major limitation. First, although
the distinction between transitory and permanent nature of earnings fluctuations is important, the crucial issue from a welfare perspective is how insurable these transitory shocks are. If they are not easily insurable, in terms of individual welfare there is little difference between the sources of inequality. In our model consumption inequality cannot be analyzed: given the assumption of perfect financial markets, all the inequality is insurable. Whether these shocks are fully insurable or not in reality is difficult to say, although to the extent to which they are transitory, there should be various ways for workers to partially insure against them (such as precautionary savings, intra-family transfers, formal borrowing, unemployment insurance, etc.). Further work on models with incomplete insurance possibilities is necessary to examine how these temporary shocks translate into consumption inequality.

Second, the probability of being adaptable $\sigma_i$ is taken as exogenous in our paper. It is likely that workers exert some control on their degree of adaptability to new technologies, for example through their investments in formal education (Nelson and Phelps, 1966). In a companion paper (Aghion, Howitt and Violante 2001), we develop a two-period OLG model where we show how young workers respond to an increase in the generality of the technology by raising their expected adaptability through their educational choice. Although the general equilibrium comparative statics of inequality with respect to $\tau$ and $\gamma$ are mitigated by the presence of this additional education channel, a simple quantitative exercise shows that with plausible parametrizations they are not reversed.
References


[9] Blundell Richard, and Ian Preston (1999); “Inequality and Uncertainty: Short-Run Uncertainty and Permanent Inequality in the US and Britain”, mimeo UCL.


Figure 1: Relative Supply and Demand in the benchmark economy
Figure 2: Within-Group Inequality as a function of $\gamma$ and $\tau$
Figure 3: Sensitivity analysis in the benchmark model
Figure 4: Relative Supply and Demand in the economy with retooling
Figure 5: Relative Supply and Demand in the economy with retooling and reviving